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Equilibrium Equity Price with Optimal Dividend Policy under Jump-Diffusion Processes*

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Abstract

This paper proposes an equilibrium model for evaluating equity with optimal dividend policy in a jump-diffusion market. In this model, a representative investor having power utility over a total consumption process evaluates the equity as the expected value of the discounted dividends with his stochastic discount factor, while a firm paying the dividends from his own cash reserve manages to maximize the equity price. This situation is formulated as a singular stochastic control problem of jump-diffusion processes. We solve this problem and give the equilibrium equity price and the optimal dividend policy. Numerical examples show that the total consumption process and the investor's risk aversion have a significant impact on the equity price and the dividend policy.

Keywords: equilibrium equity price, optimal dividend policy, risk aversion, total consumption, singular stochastic control problem, jump-diffusion process

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1 Introduction

According to a standard theory of asset pricing, equity as a financial asset is evaluated as the expected value of the discounted dividend stream with a representative investor's stochastic discount factor. Because the stochastic discount factor composed of marginal utility over total consumption puts together information about the risk aversion of the investor and the total consumption, it is one of the most important components for equity pricing. This approach is sometimes called the *consumption-based asset pricing*. For example, see Cochrane [2005], Pennacchi [2007], and Back [2010] as standard textbooks for the asset pricing theory. As a recent work, Eraker [2008] studies a consumption-based asset pricing model based on the Epstein-Zin preference (Epstein and Zin [1989]) under the assumption that macro growth rates and dividend rates follow affine processes. Martin [2013a] and Yamazaki [2014] extend the Lucas tree model originally developed by Lucas [1978] to multiple asset versions. In the Lucas tree model, all the dividends are immediately consumed by the representative investor. Martin [2013b] investigates a consumption-based asset pricing model in which the cumulant generating functions of consumption growth rates and dividend rates are assumed to be known. What seems to be lacking in the past research mentioned above is to describe the background of generating dividends. That is to say, the dividend processes have always been given exogenously.

Miller and Modigliani [1961] claimed that dividend payment policy is irrelevant to firm value in perfect markets. However, there is a lot of evidence that dividend payment policy has a huge impact on firm's equity value and it concludes that real markets are imperfect. A number of researchers succeeding to Miller and Modigliani [1961] have investigated optimal dividend policy for maximizing equity value by applying stochastic control techniques. In particular, some of their results have been remarkable contribution to the field of stochastic control. For instance, Rander and Shepp [1996], Asmussen and Taksar [1997], and Taksar and Zhou [1998], Asmussen et al. [2000], and Choulli et al. [2003] assume that a firm's cash reserve process follows a diffusion model and solve the optimal divided policy by applying the theory of singular stochastic control. Jeanblanc and Shiryaev [1995] and Cadenillas et al. [2007] formulate the optimal dividend policy as a stochastic impulse control problem. Décamps and Villeneuve [2007] analyze the interaction between the optimal dividend policy and the decision on investment in a growth opportunity of a firm, which is characterized as a mixed singular control/optimal stopping problem. Taksar [2000] surveys stochastic control models for optimal dividend policy. The problem in the past research is that the investor evaluating the equity is nearly assumed to be risk-neutral. That is, little attention has been given to the investor's risk aversion and macroeconomic factors such as total consumption.

The purpose of this study is to consider the both optimal behaviors of the firm as an issuer and the representative investor as a buyer simultaneously. On the one hand, the firm issuing the equity manages to pay dividends for maximizing the equity price. Namely, the firm tries to find optimal dividend policy. On the other hand, the representative investor that maximizes his expected utility over a total consumption stream evaluates the equity. Consequently, the equity price and the dividend policy are expected to have some implications from the risk aversion and the total consumption. More concretely, it is assumed that the representative investor has power utility and evaluates the equity with his stochastic discount factor, while the firm takes an optimal dividend payment policy to maximize the equity price evaluated by the investor. Therefore, it can be said that the equity price induced by our model is properly an equilibrium price. In addition, we are concerned with negative surprise impacts of the firm's cash reserve and the total consumption on the equity price and the dividend payment. For this purpose, it is assumed that the cash reserve process and the total consumption process follow jump-diffusion processes with only negative jumps. As a result, we formulate our optimal dividend and equity pricing problem as a stochastic singular control problem under a two-dimensional jump-diffusion process. Then, we apply the verification theorems developed by Øksendal and Sulem [2007] to solve the problem and provide the optimal dividend policy and the equilibrium equity price as the solution.

The rest of this paper is organized as follows: Section 2 introduces an equilibrium model that the representative investor evaluates the equity price with his stochastic discount factor and the firm pays dividends from his cash reserve subject to maximizing the equity price. In addition, a technical assumption ensuring the existence of the solutions to our problem is given. In Section 3, the equilibrium equity price and the optimal dividend policy are presented. It is the main theorem of this paper. Section 4 provides two simple and explanatory examples. One of them is the case that the total consumption rate and the firm's cash reserve are governed by drifted Brownian motions. Another is the case that they follow drifted Poisson processes. Concluding remarks are made in Section 5. The proof of the main theorem is placed in Appendix, which is a quite important part of this paper.

2 Model

We start with a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ which describes uncertainty of an equity market. There is a representative investor in the market that has power utility over a total consumption process $(C_t)_{t\geq 0}$. The investor's expected utility is given by

$$\mathbb{E}\left[\int_0^\infty e^{-\delta t}\frac{C_t^{1-\gamma}}{1-\gamma}dt\right],$$

where the rate of time preference δ and the relative risk aversion γ are positive constants, and $\mathbb{E}[\,\cdot\,]$ denotes an unconditional expectation with respect to $\mathbb{P}$. On the other hand, a firm pays a non-negative dividend from his cash reserve at each time.

Let $(D_t)_{t\geq 0}$ denote the firm's cumulative dividend process that is a non-decreasing $(\mathcal{F}_t)_{t\geq 0}$-adapted process and the control policy of the firm. Suppose that the total consumption process is governed by a geometric Lévy process,

$$dC_t = C_t\left(\mu_c dt + \sigma_c dW_t^1 + \alpha_c\int_{\mathbb{R}} z_1\tilde{N}^1(dt, dz_1)\right), \quad C_0 = c, \tag{2.1}$$

and the cash reserve process $(V_t^D)_{t\geq 0}$, which is the controlled process for the firm, is described by

$$dV_t^D = \mu_v dt + \sigma_v\left(\rho dW_t^1 + \sqrt{1-\rho^2}dW_t^2\right) + \sum_{k=1}^2 \alpha_{vk}\int_{\mathbb{R}} z_k\tilde{N}^k(dt, dz_k) - dD_t, \quad V_0 = v, \tag{2.2}$$

where $\alpha_c, \alpha_{v1}, \alpha_{v2} \in \mathbb{R}$, $c, v > 0$, $\mu_c, \mu_v, \sigma_c, \sigma_v \geq 0$ and $\rho \in [0,1]$ are some constants. Here, $(W_t^i)_{t\geq 0}$, $i = 1, 2$, denote one-dimensional Brownian motions and $\tilde{N}^k(dt, dz_k) := N^k(dt, dz_k) - \nu_k(dz_k)dt$, $k = 1, 2$, denote compensated Poisson random measures with Lévy measures $\nu_k(dz_k)$ such that

$$\int_{|z_k|\geq 1} |z_k|\nu_k(dz_k) < \infty,$$

and $\alpha_c z_1 \in (-1, 0]$, $\alpha_{vk}z_k \leq 0$ a.a. ν_1. These conditions mean that all the jumps give negative effects in the jump-diffusion economy. The Brownian motions and the compensated Poisson random measures are assumed to be independent of each other.

The firm bankrupts as soon as his cash reserve becomes empty. The bankruptcy time of the firm is defined as the stopping time $\tau := \inf\{t > 0 : V_t^D \leq 0\}$. According to a standard theory of asset pricing (see Cochrane [2005], Pennacchi [2007], and Back [2010] as excellent monographs for asset pricing theory), the rational investor evaluates the current price of the firm's equity P as

$$P = \mathbb{E}\left[\int_0^\tau e^{-\delta t}\left(\frac{C_t}{C_0}\right)^{-\gamma} dD_t\right]. \tag{2.3}$$

Note that the term $e^{-\delta t}(C_t/C_0)^{-\gamma}$ in the above equation is the stochastic discount factor of the representative investor having power utility with the relative risk aversion γ. The equation (2.3) means that the equity price is the expected value of the discounted dividends paid until the firm bankrupts with the stochastic discount factor. When $\gamma = 0$, the investor is risk-neutral that has been a standard assumption in past studies on optimal dividend policy problems. In this case, the problem becomes simpler, but the total consumption process is completely ignored when determining the equity price and the dividend policy. We are concerned with an impact of the investor's risk aversion and the total consumption on the market.

In order to ensure the existence of the optimal dividend policy and the equilibrium equity price, the following parameter restriction is imposed.

Assumption 1 *The model parameters satisfy the inequality*

$$\delta > -\gamma\mu_c + \frac{1}{2}\gamma(2\gamma+1)\sigma_c^2 + \int_{\mathbb{R}}\left\{(1+\alpha_c z_1)^{-2\gamma} - \frac{1}{2} + \gamma\alpha_c z_1\right\}\nu_1(dz_1). \tag{2.4}$$

Note that if the inequality (2.4) holds then the model parameters satisfy

$$\delta > -\gamma\mu_c + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + \int_{\mathbb{R}}\left\{(1+\alpha_c z_1)^{-\gamma} - 1 + \gamma\alpha_c z_1\right\}\nu_1(dz_1). \tag{2.5}$$

The inequality (2.5) is a sufficient condition that the characteristic equation derived in the next section has both the positive and negative roots.

The firm tries to find the optimal dividend payment policy to maximize the equity price (2.3). Equivalently, we seek $(D_t^*)_{t\geq 0} \in \mathcal{A}$ and $J^*(s,c,v)$ such that

$$J^*(s,c,v) := \sup_{D\in\mathcal{A}} J^{(D)}(s,c,v) = J^{(D^*)}(s,c,v), \tag{2.6}$$

where $\mathcal{A}$ denotes the set of all admissible control processes $(D_t)_{t\geq 0}$ that are non-decreasing and right-continuous processes and for $s \geq 0$, we define

$$J^{(D)}(s,c,v) := \mathbb{E}\left[\int_0^\tau e^{-\delta(s+t)} C_t^{-\gamma} dD_t\right]. \tag{2.7}$$

The formulations (2.6) and (2.7) can be regarded as a stochastic singular control problem under a two-dimensional jump-diffusion process $(C_t, V_t^D)_{t\geq 0}$.

3 Optimal Dividend Policy and Equity Price

In this section, the optimal dividend policy and the equilibrium equity price are provided as the solution to the singular stochastic control problem defined by (2.6) and (2.7). Theorem

1 presented below is the main result of this paper, while the proof of Theorem 1 is placed in Appendix because long discussions including the verification steps for the solution to the integro-variational inequality are needed. Before presenting Theorem 1, two technical lemmas are given.

Lemma 1 *For $a = (a_+, a_-) \in \mathbb{R}^2$ such that $a_- < 0 < a_+$ and $|a_+| \le |a_-|$, define the function $\Psi_a : \mathbb{R}_+ \to \mathbb{R}$ by*

$$\Psi_a(x) = e^{a_+ x} - e^{a_- x}. \tag{3.1}$$

Then, the following statements are satisfied.

1. *Ψ_a is non-negative and strictly increasing with $\Psi_a(0) = 0$.*
2. *Ψ_a is concave for $0 \le x < x^*$ while it is convex for $x^* < x$, where*

$$x^* = \frac{1}{a_+ - a_-} \ln \left(\frac{a_-}{a_+} \right)^2 .$$

Proof of Lemma 1: The statement 1 is trivial. Note that x^* is a unique root of $\Psi_a''(x) = 0$. Therefore, the statement 2 is also trivial. □

Define the characteristic equation as $\eta(x) = 0$, where

$$\begin{aligned} \eta(x) \; := \; & -\delta - \gamma\mu_c + \mu_v x + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + \frac{1}{2}\sigma_v^2 x^2 - \gamma\rho\sigma_c\sigma_v x \\ & + \int_{\mathbb{R}} \left\{ (1+\alpha_c z_1)^{-\gamma} e^{\alpha_{v1} z_1 x} - 1 + \gamma\alpha_c z_1 - \alpha_{v1} z_1 x \right\} \nu_1(dz_1) \\ & + \int_{\mathbb{R}} \left\{ e^{\alpha_{v2} z_2 x} - 1 - \alpha_{v2} z_2 x \right\} \nu_2(dz_2). \end{aligned} \tag{3.2}$$

The coefficients of the function $\eta(x)$ are composed of the parameters of the total consumption and the cash reserve including their Lévy measures and correlation, the rate of time preference, and the risk aversion. This means that the characteristic equation is characterized not only by the cash reserve process but also by a total condition of the market. However, note that just the initial consumption c is excluded from the equation.

Lemma 2 *The characteristic equation $\eta(x) = 0$ has two roots q_+, q_- such that $q_- < 0 < q_+$.*

Proof of Lemma 2: By the assumption (2.5), we have $\eta(0) < 0$. Because $\alpha_c z_1 > -1$ and $\alpha_{v1} z_1 \le 0$ a.a. ν_1 and $\alpha_{v2} z_2 \le 0$ a.a. ν_2, we have $\lim_{x \to \pm\infty} \eta(x) = \infty$. □

Theorem 1 *Let $q = (q_+, q_-) \in \mathbb{R}$ be a vector of the two roots of the characteristic equation $\eta(x) = 0$ such that $q_- < 0 < q_+$. Define $K_q = \Psi_q'(0)/\Psi_q'(v^*)$, where*

$$v^* = \frac{1}{q_+ - q_-} \ln \left(\frac{q_-}{q_+} \right)^2 . \tag{3.3}$$

If the model parameters satisfy the inequality

$$\delta > -\gamma\mu_c + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + \int_{\mathbb{R}} \left\{ K_q (1+\alpha_c z_1)^{-\gamma} - 1 + \gamma\alpha_c z_1 \right\} \nu_1(dz_1) + (K_q - 1)\nu_2(\mathbb{R}), \tag{3.4}$$

then the optimal dividend payment process $(D_t^*)_{t\geq 0}$ *is given by*

$$D_t^* = \max\left(0, \sup_{0\leq s\leq t}(V_s^0 - v^*)\right), \tag{3.5}$$

and the equilibrium equity price P^* *is given by*

$$P^* = \mathbb{E}\left[\int_0^\tau e^{-\delta t}\left(\frac{C_t}{C_0}\right)^{-\gamma} dD_t^*\right] = \frac{\Psi_q(v)}{\Psi_q'(v^*)}, \qquad \textit{for} \quad 0 \leq v \leq v^*. \tag{3.6}$$

The value v^* defined by (3.3) can be interpreted as the optimal dividend policy for the firm. Thus, the firm does not pay any dividends when $V_t^{D^*} \in (0, v^*)$. On the contrary, if $V_t^{D^*} \geq v^*$, the firm continues to pay dividends until $V_t^{D^*} < v^*$. Therefore, the optimal controlled cash reserve process $(V_t^{D^*})_{t\geq 0}$ becomes a *reflected jump-diffusion process* back into the interval $(0, v^*)$. Because $\Psi_q(x)$ is an increasing function, the equilibrium equity price has the upper and lower bounds,

$$0 \;\leq\; P^* \;\leq\; \frac{\Psi_q(v^*)}{\Psi_q'(v^*)}.$$

It is interesting that the optimal dividend payment (3.5) and the equilibrium equity price (3.6) seem the same formulas as the case that the investor is risk-neutral, that is, $\gamma = 0$. For example, see the equations (2.4) and (2.5) in Décamps and Villeneuve [2007] for the risk-neutral equity pricing formula. However, information about the total consumption process and the investor's risk aversion as well as the firm's cash reserve process is integrated into the roots of the characteristic equation q_+ and q_-. Note that the initial consumption c does not affect the dividend policy and the equity price, while other parameters of the total consumption process are significant elements to determine the roots.

The optimal dividend payment process $(D_t^*)_{t\geq 0}$ defined by (3.5) is the local time of the sifted cash reserve process $(V_t^0 - v^*)_{t\geq 0}$ at the maximum. In the context of Lévy processes, the spectrally negative Lévy process $(V_t^0 - v^*)_{t\geq 0}$ is *creeping* at the maximum. The local time (3.5) is the same form as the local time of a drifted Brownian motion at the maximum because the maximum process of $(V_t^0 - v^*)_{t\geq 0}$ is continuous due to no positive jumps.

The inequality (3.4) is needed as a sufficient condition to prove the verification of the solution to the singular stochastic control problem. The condition (3.4) is occasionally stronger than the assumption (2.5).

4 Examples

This section gives two simple examples to understand the equilibrium model proposed above. One of them is the case that the total consumption process and the cash reserve process are driven by diffusion processes without any jumps. Another example is the case that both of them are governed by drifted negative jump processes without Brownian motions. We illustrate an impact of macroeconomic factors out of the firm on the equilibrium equity price and the optimal dividend policy. All the numerical results are computed by Matlab R2015a.

4.1 Diffusion Market

Consider the no jump component case, that is, $\lambda_1 = \lambda_2 = \alpha_c = \alpha_{v1} = \alpha_{v2} = 0$. In this case, the roots of the characteristic equation $\eta(x) = 0$ are

$$q_+ = \frac{g_1 + \sqrt{g_2}}{\sigma_v^2} \qquad \text{and} \qquad q_- = \frac{g_1 - \sqrt{g_2}}{\sigma_v^2},$$

where we define

$$\begin{aligned} g_1 &:= g_1(\sigma_c, \mu_v, \sigma_v, \rho, \gamma) := \gamma\rho\sigma_c\sigma_v - \mu_v, \\ g_2 &:= g_2(\mu_c, \sigma_c, \mu_v, \sigma_v, \rho, \gamma) := g_1^2 + 2\delta\sigma_v^2 + 2\gamma\mu_c\sigma_v^2 - \gamma(\gamma+1)\sigma_c^2\sigma_v^2. \end{aligned}$$

Note that $\sqrt{g_2} + g_1 > 0$ and $\sqrt{g_2} - g_1 > 0$. As a result, the explicit expressions of the optimal dividend policy and the equilibrium equity price are

$$v^* = \frac{\sigma_v^2}{\sqrt{g_2}} \ln\left(\frac{\sqrt{g_2} - g_1}{\sqrt{g_2} + g_1}\right),$$

and

$$P^* = \frac{\sigma_v^2}{2\sqrt{g_2}}\left(\frac{\sqrt{g_2} + g_1}{\sqrt{g_2} - g_1}\right)^{g_1/\sqrt{g_2}}\left\{\exp\left(\frac{g_1 + \sqrt{g_2}}{\sigma_v^2} v\right) - \exp\left(\frac{g_1 - \sqrt{g_2}}{\sigma_v^2} v\right)\right\},$$

respectively. The upper bound of the equity price is given by $2\sigma_v^2 g_1/(g_1^2 - g_2)$.

Next, numerical examples of the diffusion case are presented as follows. The benchmark parameters are listed on Table 1 which implies that the representative investor has log utility. The equilibrium equity price equity price P^* and the optimal dividend policy v^* with the benchmark parameters are 152.3858 and 79.8262, respectively. In order to investigate an impact of macroeconomic factors on the dividend policy and the equity price, we recalculate equilibrium equity prices and optimal dividend policies by changing the value of each parameter μ_c, σ_c, ρ, and γ. Recall that these four parameters are external factors of the firm and have never been treated explicitly in past literature. All the combinations of parameter values in our numerical example are set to satisfy Assumption 1. Note that in the diffusion case if Assumption 1 holds then the condition (3.4) in Theorem 1 is satisfied automatically. Figures 1-4 depict the equity prices and the dividend policies. It is found from the results that the equilibrium equity price and the optimal dividend policy highly depend not only on the cash flow generated by the firm but also on the total consumption and the risk aversion of the representative investor. It is worthwhile noting that when the investor is risk-neutral, that is $\gamma = 0$, the equilibrium equity price is 187.9651 and the optimal dividend policy is 84.5949. As shown in Figure 4, the risk-neutral price is trivially overestimated in comparison with the risk-averse prices and the firm pays less amount of dividends to the risk-neutral investor than the risk-averse investor.

Table 1: Benchmark parameters for the diffusion case

μ_c	σ_c	μ_v	σ_v	ρ	γ	δ	v
0.02	0.05	30	30	0.5	1.0	0.1	20

4.2 Jump Market

The next example is a jump market in the absence of Brownian motions. Put $\sigma_c = \sigma_{v1} = \sigma_{v2} = 0$ and $\alpha_c, \alpha_{v1}, \alpha_{v2} < 0$, and define the Lévy measures as

$$\nu_1(dz_1) = \lambda_1 \delta(z_1 - 1)dz_1 \qquad \text{and} \qquad \nu_2(dz_2) = \lambda_2 \delta(z_2 - 1)dz_2,$$

where $\lambda_1, \lambda_2 > 0$ are the jump intensities and $\delta(x)$ denotes the Dirac delta function, that is, the jumps are governed by Poisson processes. In this case, the characteristic equation $\eta(x) = 0$ can be written as

$$(\mu_v - \lambda_1 \alpha_{v1} - \lambda_2 \alpha_{v2})x + \lambda_1 (1 + \alpha_c)^{-\gamma} e^{\alpha_{v1} x} + \lambda_2 e^{\alpha_{v2} x} = \delta + \gamma \mu_c + \lambda_1 - \lambda_1 \gamma \alpha_c + \lambda_2. \quad (4.1)$$

Although this is the simplest example among jump processes, the closed-form solutions to the equation (4.1) is not able to be obtained unfortunately. Nevertheless, it is easy to obtain the solutions numerically, because we know that the characteristic equation has negative and positive roots under Assumption 1. In our numerical example, Matlab function "fzero" is used to obtain the numerical solutions to the characteristic equation (4.1).

Next, numerical examples of the jump case are provided as follows. The benchmark parameters are listed on Table 2 which also implies that the investor has log utility. The equilibrium equity price P^* and the optimal dividend policy v^* with the benchmark parameters are 21.7695 and 25.5148, respectively. Similarly to the diffusion case, we recalculate equilibrium equity prices and optimal dividend policies by changing the value of each parameter $\mu_c, \alpha_c, \lambda_1$, and γ, all of which are also external factors of the firm. All the combinations of parameter values are set to satisfy both Assumption 1 and the condition (3.4). Figures 5-8 plot the equity prices and the dividend policies. The results make it clear that the total consumption and the risk aversion are quite important to determine the optimal dividend policy and the equilibrium equity price. Incidentally, when the investor is risk-neutral, the equilibrium equity price is 23.1326 which is obviously overestimated and the optimal dividend policy is 32.7801. See Figure 8.

Table 2: Benchmark parameters for the jump case

μ_c	α_c	μ_v	α_{v1}	α_{v2}	λ_1	λ_2	γ	δ	v
0.02	-0.03	5	-20	-50	0.1	0.1	1.0	0.1	20

5 Concluding Remarks

We propose an equilibrium model for evaluating equity price in a jump-diffusion market. In this model, a representative investor with power utility over a total consumption stream evaluates the equity price as the expected value of the discounted dividends with his stochastic discount factor. A firm issuing the equity pays the dividend at each time from his cash reserve subject to maximizing the equity price. It is assumed that the dynamics of the firm's cash reserve are described by a controlled Lévy process and the total consumption is governed by a geometric Lévy process. The both processes are assumed to have only negative jumps.

The pricing problem proposed in this paper is formulated as a singular stochastic control problem of a two-dimensional jump-diffusion process. By applying the verification theorems in Øksendal and Sulem [2007], the equilibrium equity price and the optimal dividend policy are obtained as the solutions to the problem. The optimal dividend payment is given by a local time of the shifted cash reserve process with no dividend payments at the maximum. It is shown that information about the investor's risk preference and uncertainty of the total consumption as well as the cash reserve is compressed into the roots of the characteristic equation that essentially determines the equilibrium equity price and the optimal dividend policy.

As an explanatory example, we present the closed-form expressions of the optimal dividend policy and the equilibrium equity in a diffusion market without any jumps. In the the case of pure jump processes, the solutions are obtained numerically because a numerical method is needed to solve the characteristic equation. Numerical examples demonstrate that the total consumption process and the investor's risk aversion, which have been ignored in past literature, have a significant impact on the equilibrium equity price and the optimal dividend policy.

Finally, a further direction for future study will be empirical analysis based on the model we proposed. In order to achieve it, we have to establish an estimation method for the model parameters. Needless to say, sufficient data series are necessary.

A General Formulation of Singular Control Problem

Following to the chapter 6 of Øksendal and Sulem [2007], we briefly review a general formulation of singular stochastic control problems with jump-diffusion processes.

Suppose that a controlled process $(Y_t^\xi)_{t\geq 0}$ taking values in $\mathbb{R}^3$ is described by the SDE,

$$dY_t^\xi = \mu(Y_t^\xi)dt + \sigma(Y_t^\xi)dW_t + \int_{\mathbb{R}^2} \alpha(Y_{t-}^\xi, z)\tilde{N}(dt, dz) + \kappa(Y_{t-}^\xi)d\xi_t, \quad Y_0^\xi = y \in \mathbb{R}^3, \tag{A.1}$$

where $\mu = [\mu_n] : \mathbb{R}^3 \to \mathbb{R}^3$, $\sigma = [\sigma_{ni}] : \mathbb{R}^3 \to \mathbb{R}^{3\times 2}$, and $\alpha = [\alpha_{nk}] : \mathbb{R}^3 \times \mathbb{R}^2 \to \mathbb{R}^{3\times 2}$ satisfy the linear growth condition and the Lipschitz condition, and $\kappa = [\kappa_n] : \mathbb{R}^3 \to \mathbb{R}^3$ is a continuous function. Here, $(W_t)_{t\geq 0}$ is a two-dimensional Brownian motion and

$$\tilde{N}(dt, dz) := (N^1(dt, dz_1) - \nu_1(dz_1)dt, N^2(dt, dz_2) - \nu_2(dz_2)dt)^\top$$

is a two-dimensional compensated Poisson random measure with Lévy measures ν_k satisfying

$$\int_{|z_k|\geq 1} |z_k|\nu_k(dz_k) < \infty,$$

for $k = 1, 2$. The Brownian motion is independent of the compensated Poisson random measure. A control process $(\xi_t)_{t\geq 0}$ taking values in $\mathbb{R}$ is assumed to be a $(\mathcal{F}_t)_{t\geq 0}$-adapted, non-decreasing, and right-continuous process with $\xi_{0-} = 0$.

Suppose that we are given an object functional $J^{(\xi)}(y)$ of the form

$$J^{(\xi)}(y) = \mathbb{E}^y\left[\int_0^\tau f(Y_{t-}^\xi)d\xi_t\right].$$

Here, $f : \mathbb{R}^3 \to \mathbb{R}$ are a given continuous function and $\tau = \inf\{t > 0 : Y_t^\xi \notin \mathcal{S}\}$ is the bankruptcy time, where $\mathcal{S} \subset \mathbb{R}^3$ is a given *solvency set*. The set of all admissible controls denoted by $\mathcal{A}$ contains control processes $(\xi_t)_{t\geq 0}$ such that the SDE (A.1) has a unique strong solution and

$$\mathbb{E}^y\left[\int_0^\tau |f(Y_{t-}^\xi)|d\xi_t\right] < \infty. \tag{A.2}$$

The objective is to find the value function $J^*(y)$ and the optimal control $(\xi_t^*)_{t\geq 0}$ such that

$$J^*(y) := \sup_{\xi\in\mathcal{A}} J^{(\xi)}(y). \tag{A.3}$$

Figure 1: Drift of total consumption (diffusion case)

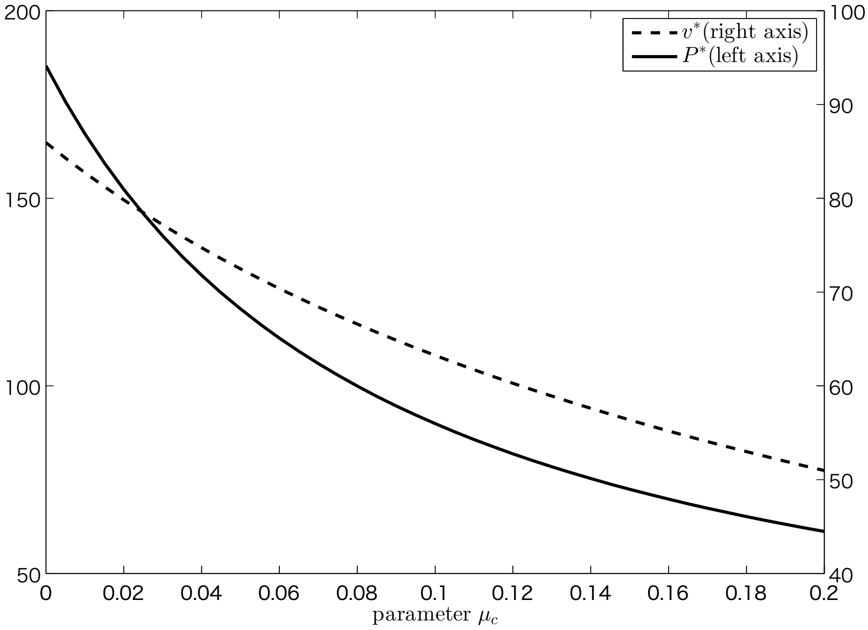


Figure 2: Volatility of total consumption (diffusion case)

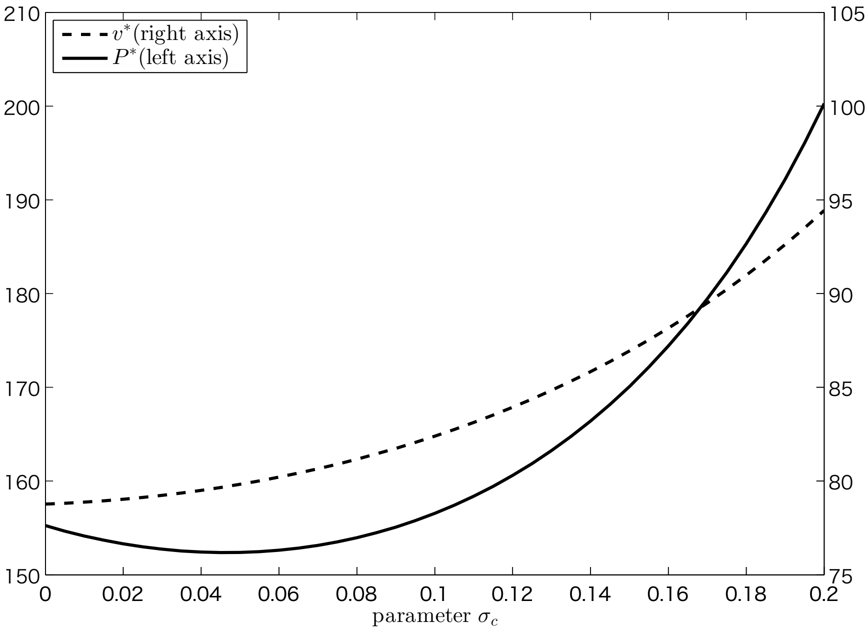


Figure 3: Correlation (diffusion case)

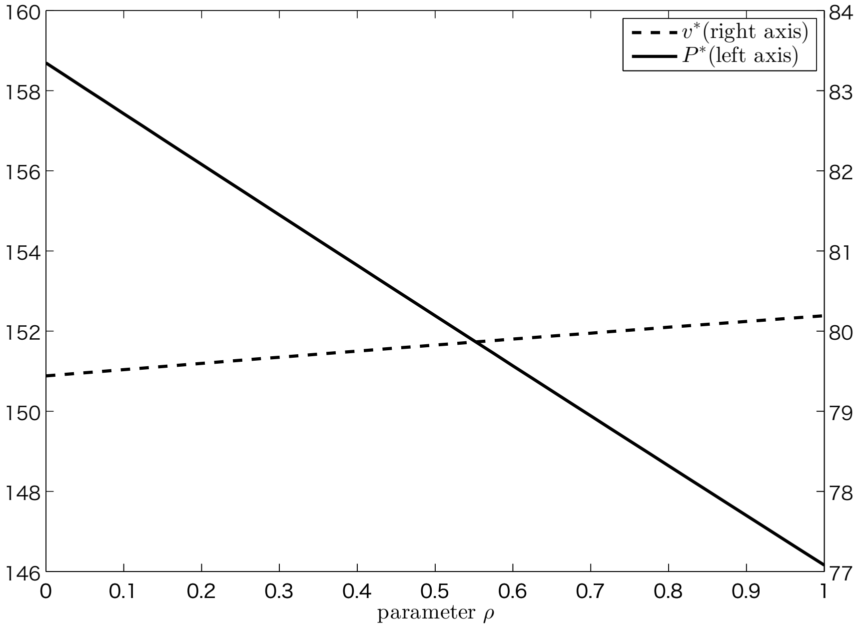


Figure 4: Risk aversion (diffusion case)

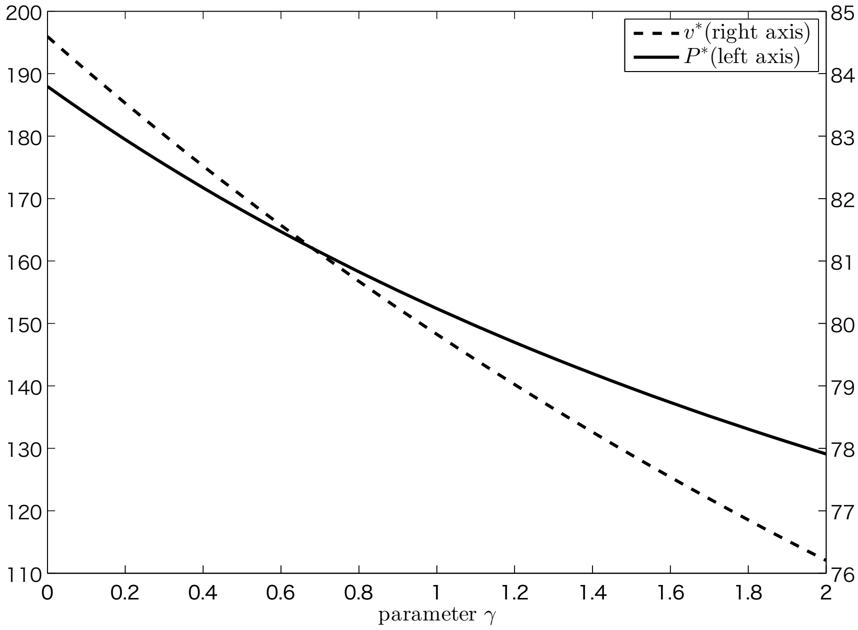


Figure 5: Drift of total consumption (jump case)

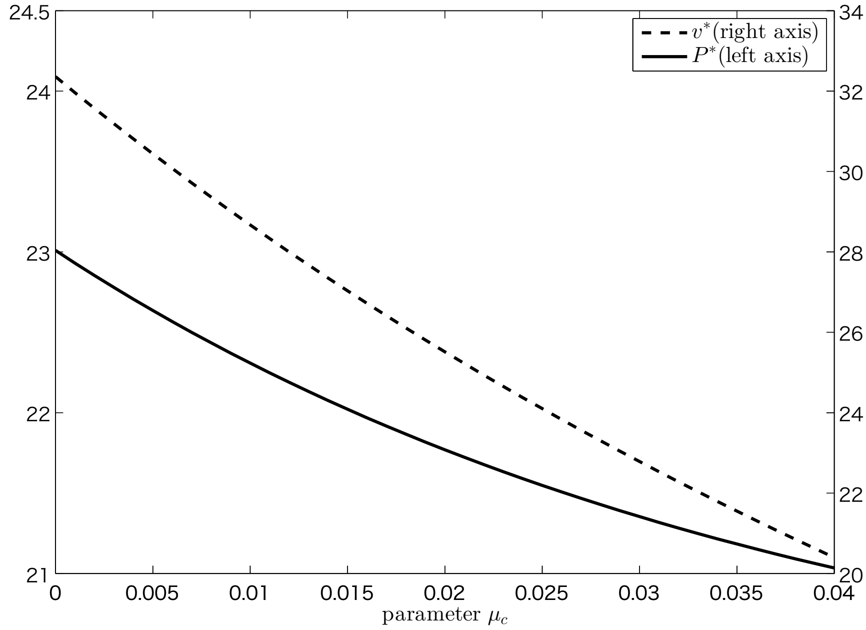


Figure 6: Jump size of total consumption (jump case)

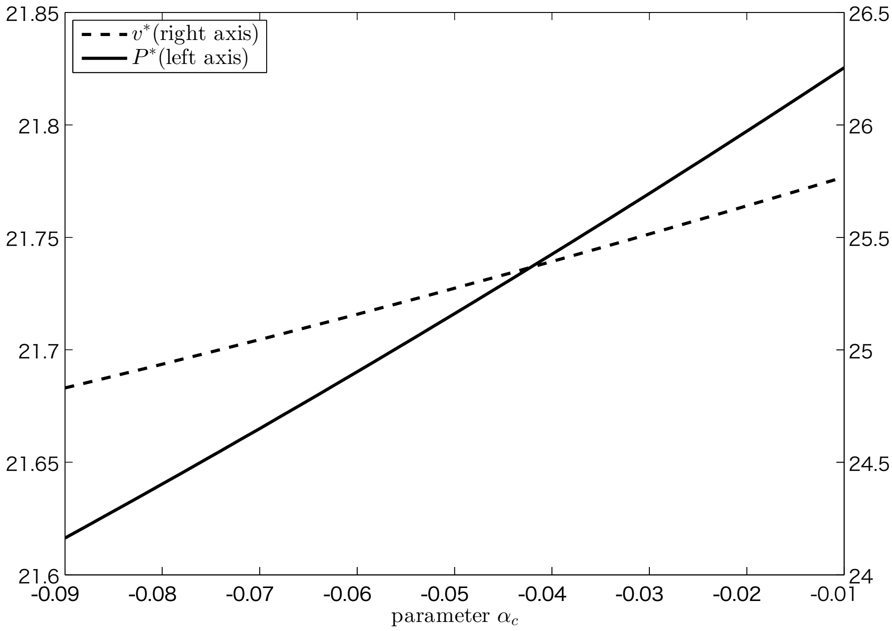


Figure 7: Jump intensity of common jumps (jump case)

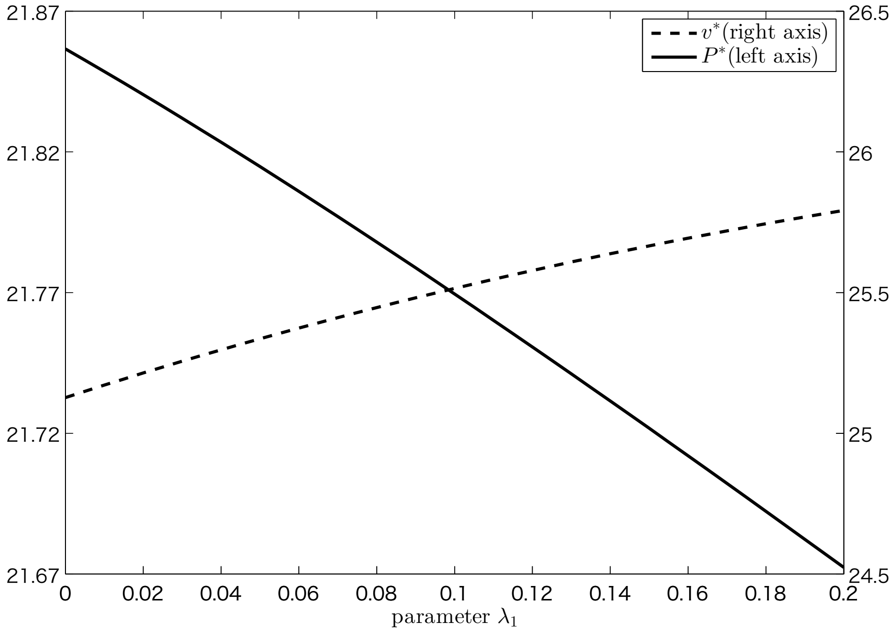
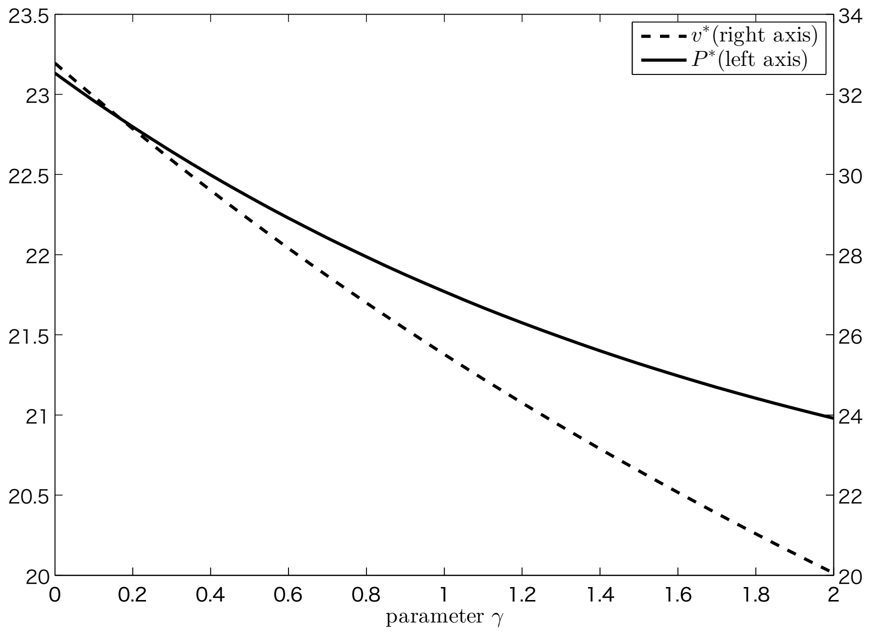


Figure 8: Risk aversion (jump case)



Next, the infinitesimal generator $\mathcal{G}$ of the jump-diffusion process $(Y_t^0)_{t\geq 0}$ is defined as

$$\begin{aligned}\mathcal{G}\phi(y) &= \sum_{n=1}^{3}\mu_n(y)\frac{\partial\phi}{\partial y_n}(y)+\frac{1}{2}\sum_{m,n=1}^{3}(\sigma\sigma^\top)_{mn}(y)\frac{\partial^2\phi}{\partial y_m\partial y_n}(y)\\ &\quad+\sum_{k=1}^{2}\int_{\mathbb{R}}\left\{\phi(y+\alpha^{(k)}(y,z_k))-\phi(y)-\nabla\phi(y)^\top\alpha^{(k)}(y,z_k)\right\}\nu_k(dz_k), \end{aligned}\tag{A.4}$$

where $\phi\in C^2(\mathrm{Int}(\mathcal{S}))$ and $\alpha^{(k)}\in\mathbb{R}^n$ denotes the k-th column of the $n\times k$ matrix $\alpha=[\alpha_{nk}]$.

The following propositions are slightly simplified versions of the verification theorem by Øksendal and Sulem [2007] to fit our problem. The proofs of the propositions can be found in the theorem 6.2 of Øksendal and Sulem [2007] or in the theorem 2.2 of Framstad et al. [2001].

Proposition 1 *(Theorem 6.2-(a) in Øksendal and Sulem [2007]) Suppose that there exists a function* $\phi\in C^2(\mathrm{Int}(\mathcal{S}))\cap C(\bar{\mathcal{S}})$ *such that*

(i) $\mathcal{G}\phi(y)\leq 0$ *for all* $y\in\mathcal{S}$.

(ii) $\displaystyle\sum_{n=1}^{3}\kappa_n(y)\frac{\partial\phi}{\partial y_n}(y)+f(y)\leq 0$ *for all* $y\in\mathcal{S}$.

(iii) For all $\xi\in\mathcal{A}$,

$$\mathbb{E}^y\left[\int_0^\tau|\sigma^\top(Y_t^\xi)\nabla\phi(Y_t^\xi)|^2dt\right]<\infty,$$

and

$$\mathbb{E}^y\left[\sum_{k=1}^{2}\int_0^\tau\int_{\mathbb{R}}|\phi(Y_t^\xi+\alpha^{(k)}(Y_t^\xi,z_k))-\phi(Y_t^\xi)|^2\nu_k(dz_k)dt\right]<\infty.$$

(iv) $\displaystyle\lim_{t\to\tau-}\phi(Y_t^\xi)=0$ *a.s., for all* $\xi\in\mathcal{A}$.

(v) $(\phi^-(Y_t^\xi))_{t\leq\tau}$ *is uniformly integrable for all* $\xi\in\mathcal{A}$ *and* $y\in\mathcal{S}$.

Then

$$\phi(y)\geq J^*(y),\qquad \text{for all }\ y\in\mathcal{S}.\tag{A.5}$$

Proposition 2 *(Theorem 6.2-(b) in Øksendal and Sulem [2007]) Let* $\phi\in C^2(\mathrm{Int}(\mathcal{S}))\cap C(\bar{\mathcal{S}})$. *Define the non-intervention region* $\mathcal{D}$ *by*

$$\mathcal{D}=\left\{y\in\mathcal{S}:\sum_{n=1}^{3}\kappa_n(y)\frac{\partial\phi}{\partial y_n}(y)+f(y)<0\right\}.$$

In addition to the conditions (i)-(v) in Proposition 1, suppose that

(vi) $\mathcal{G}\phi(y)=0$ *for all* $y\in\bar{\mathcal{D}}$.

Moreover, suppose that there exists $\xi^*\in\mathcal{A}$ *such that* *(vii)* $Y_t^{\xi^*} \in \bar{\mathcal{D}}$ *for every* $t \geq 0$.

(viii) $\left\{ \sum_{n=1}^{3} \kappa_n (Y_{t-}^{\xi^*}) \frac{\partial \phi}{\partial y_n} (Y_{t-}^{\xi^*}) + f(Y_{t-}^{\xi^*}) \right\} d\xi_t^{*(c)} = 0$, *for every* $t \geq 0$. *Here,* $\xi_t^{(c)}$ *denotes the continuous part of* ξ_t.

(ix) $\Delta_{\xi^*} \phi(Y_{t_j}^{\xi^*}) + f(Y_{t_j-}^{\xi^*}) \Delta \xi_{t_j}^* = 0$ *for all jumping times* t_j *of* $(\xi_t^*)_{t \geq 0}$. *Here,* $\Delta_\xi \phi(Y_{t_j}^{\xi}) := \phi(Y_{t_j}^{\xi}) - \phi(Y_{t_j-}^{\xi} + \Delta_N Y_{t_j}^{\xi})$, *where* $\Delta_N Y_t^{\xi}$ *denotes the jumps of* $(Y_t^{\xi})_{t \geq 0}$ *caused by* $N(t, z)$.

(x) $\lim_{T \to \infty} \mathbb{E}^y \left[\phi(Y_{\tau \wedge T}^{\xi^*}) \right] = 0$.

Then

$$\phi(y) = J^*(y), \tag{A.6}$$

and $(\xi_t^*)_{t \geq 0}$ *is an optimal control.*

B Proof of Theorem 1

In order to apply Proposition 1 and 2 to our problem, we put $\xi_t = D_t$, $Y_t^D = (s + t, C_t, V_t^D)^\top$, and $y = (s, c, v)^\top$. Define the solvency set as $\mathcal{S} = [0, \infty) \times \mathbb{R}_{++}^2$, where $\mathbb{R}_{++}$ denotes the strictly positive half line. Then, $(Y_t^D)_{t \geq 0}$ follows the SDE

$$\begin{aligned} dY_t^D = \begin{bmatrix} dt \\ dC_t \\ dV_t^D \end{bmatrix} &= \begin{bmatrix} 1 \\ \mu_c C_t \\ \mu_v \end{bmatrix} dt + \begin{bmatrix} 0 & 0 \\ \sigma_c C_t & 0 \\ \sigma_v \rho & \sigma_v \sqrt{1 - \rho^2} \end{bmatrix} dW_t \\ &\quad + \int_{\mathbb{R}^2} \begin{bmatrix} 0 & 0 \\ \alpha_c z_1 C_t & 0 \\ \alpha_{v1} z_1 & \alpha_{v2} z_2 \end{bmatrix} \tilde{N}(dt, dz) + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} dD_t. \end{aligned}$$

The infinitesimal generator $\mathcal{G}$ of $(Y_t^0)_{t \geq 0}$ is given by

$$\begin{aligned} \mathcal{G}\phi(y) &= \frac{\partial \phi}{\partial s}(y) + \mu_c c \frac{\partial \phi}{\partial c}(y) + \mu_v \frac{\partial \phi}{\partial v}(y) + \frac{1}{2}\sigma_c^2 c^2 \frac{\partial^2 \phi}{\partial c^2}(y) + \frac{1}{2}\sigma_v^2 \frac{\partial^2 \phi}{\partial v^2}(y) + \sigma_c \sigma_v \rho c \frac{\partial^2 \phi}{\partial c \partial v}(y) \\ &\quad + \int_{\mathbb{R}} \left\{ \phi(s, c + \alpha_c z_1 c, v + \alpha_{v1} z_1) - \phi(y) - \alpha_c z_1 c \frac{\partial \phi}{\partial c}(y) - \alpha_{v1} z_1 \frac{\partial \phi}{\partial v}(y) \right\} \nu_1(dz_1) \\ &\quad + \int_{\mathbb{R}} \left\{ \phi(s, c, v + \alpha_{v2} z_2) - \phi(y) - \alpha_{v2} z_2 \frac{\partial \phi}{\partial v}(y) \right\} \nu_2(dz_2). \end{aligned}$$

Let $x = (c, v)^\top$. If $\phi(y) = e^{-\delta s}\psi(x)$, we have $\mathcal{G}\phi(y) = e^{-\delta s}\mathcal{G}_0\psi(x)$, where

$$\begin{aligned} \mathcal{G}_0\psi(x) &= -\delta\psi(x) + \mu_c c \frac{\partial \psi}{\partial c}(x) + \mu_v \frac{\partial \psi}{\partial v}(x) + \frac{1}{2}\sigma_c^2 c^2 \frac{\partial^2 \psi}{\partial c^2}(x) + \frac{1}{2}\sigma_v^2 \frac{\partial^2 \psi}{\partial v^2}(x) + \sigma_c \sigma_v \rho c \frac{\partial^2 \psi}{\partial c \partial v}(x) \\ &\quad + \int_{\mathbb{R}} \left\{ \psi(c + \alpha_c z_1 c, v + \alpha_{v1} z_1) - \psi(x) - \alpha_c z_1 c \frac{\partial \psi}{\partial c}(x) - \alpha_{v1} z_1 \frac{\partial \psi}{\partial v}(x) \right\} \nu_1(dz_1) \\ &\quad + \int_{\mathbb{R}} \left\{ \psi(c, v + \alpha_{v2} z_2) - \psi(x) - \alpha_{v2} z_2 \frac{\partial \psi}{\partial v}(x) \right\} \nu_2(dz_2). \end{aligned} \tag{B.1}$$

In Propositions 1 and 2, we put $f(y) = e^{-\delta s} c^{-\gamma}$. The non-intervention region $\mathcal{D}$ is given by

$$\mathcal{D} = \left\{ x \in \mathbb{R}^2_{++} : -\frac{\partial \psi}{\partial v}(x) + c^{-\gamma} < 0 \right\}. \tag{B.2}$$

Suppose that $\mathcal{D}$ has the form

$$\mathcal{D} = \{ v \in \mathbb{R}_{++} : 0 < v < v^* \}, \tag{B.3}$$

for some $v^* > 0$. We tentatively choose a function ψ of the form $\psi(x) := \psi_c(c)\psi_v(v) := c^{-\gamma} e^{qv}$ for some constant $q \in \mathbb{R}$ and substitute it into (B.1). Then the condition (vi) of Proposition 2 is reduced to the characteristic equation $\eta(q) = 0$. By Lemma 2, the characteristic equation has two solution q_+, q_- such that $q_- < 0 < q_+$.

If $v \geq v^*$, the condition (ii) of Proposition 1 requires

$$-\psi_v'(v) + 1 = 0.$$

Therefore, we redefine

$$\psi(x) := \psi_c(c)\psi_v(v) := \begin{cases} c^{-\gamma}\left(K_1 e^{q_+ v} + K_2 e^{q_- v}\right), & \text{if} \quad 0 < v < v^* \\ c^{-\gamma}\left(v + K_3\right), & \text{if} \quad v^* \leq v \end{cases} \tag{B.4}$$

where K_1, K_2, K_3 are some constants which will be determined below.

Since the condition (iv) of Proposition 1 requires $\psi_v(0) = 0$, we must have $K_2 = -K_1$. The smooth pasting condition $\phi \in C^2(\mathrm{Int}(\mathcal{S}))$ requires

$$\begin{cases} K_1 \Psi_q(v^*) = v^* + K_3 & (\text{continuity at } v^*) \\ K_1 \Psi_q'(v^*) = 1 & (\phi \in C^1 \text{ at } v^*) \\ K_1 \Psi_q''(v^*) = 0 & (\phi \in C^2 \text{ at } v^*) \end{cases} \tag{B.5}$$

From (B.5), we obtain

$$v^* = \frac{1}{q_+ - q_-} \ln\left(\frac{q_-}{q_+}\right)^2, \qquad K_1 = \frac{1}{\Psi_q'(v^*)}, \qquad K_3 = \frac{\Psi_q(v^*)}{\Psi_q'(v^*)} - v^*.$$

We have to verify that the function $\phi(y) = e^{-\delta s}\psi(x) = e^{-\delta s}\psi_c(c)\psi_v(v)$ defined by (B.4) satisfies the conditions (i)-(x) in Propositions 1 and 2. Note that $\phi(y)$ satisfies the conditions (iv) and (vi) by construction. The condition (v) is trivial because $\phi^-(Y_t^D) = 0$. In the following, we will verify the remaining conditions.

B.1 Condition (i)

Since the condition (vi) has been satisfied, we need only to show $\mathcal{G}\phi(y) \leq 0$ for $y \notin \mathcal{D}$. This is equivalent to $G(v) := c^{\gamma}\mathcal{G}_0\psi(x) \leq 0$ for $v \geq v^*$, where

$$G(v) = \left[-\delta - \gamma\mu_c + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2\right](v + K_3) + \mu_v - \gamma\rho\sigma_c\sigma_v + I_1(v) + I_2(v) + I_3(v).$$

Here, we put

$$\begin{aligned} I_1(v) &= \int_{v+\alpha_{v1}z_1<v^*} \Big\{(1+\alpha_c z_1)^{-\gamma} K_1 \Psi_q(v+\alpha_{v1}z_1) \\ &\qquad - (v+\alpha_{v1}z_1+K_3) + \gamma\alpha_c z_1(v+K_3)\Big\}\nu_1(dz_1), \\ I_2(v) &= \int_{v+\alpha_{v1}z_1\geq v^*} \Big\{(1+\alpha_c z_1)^{-\gamma}(v+\alpha_{v1}z_1+K_3) \\ &\qquad - (v+\alpha_{v1}z_1+K_3) + \gamma\alpha_c z_1(v+K_3)\Big\}\nu_1(dz_1), \\ I_3(v) &= \int_{v+\alpha_{v2}z_2\geq v^*} \Big\{K_1\Psi_q(v+\alpha_{v2}z_2) - (v+\alpha_{v2}z_2+K_3)\Big\}\nu_2(dz_2). \end{aligned}$$

Because of concavity of Ψ_q on $[0, v^*]$ and $K_q \geq 1$ by Lemma 1, we have

$$\begin{aligned} (I_1+I_2)'(v) &\leq \int_{v+\alpha_{v1}z_1<v^*} \left\{K_q(1+\alpha_c z_1)^{-\gamma} - 1 + \gamma\alpha_c z_1\right\}\nu_1(dz_1) \\ &\quad + \int_{v+\alpha_{v1}z_1\geq v^*} \left\{(1+\alpha_c z_1)^{-\gamma} - 1 + \gamma\alpha_c z_1\right\}\nu_1(dz_1) \\ &\leq \int_{\mathbb{R}} \left\{K_q(1+\alpha_c z_1)^{-\gamma} - 1 + \gamma\alpha_c z_1\right\}\nu_1(dz_1), \end{aligned}$$

and

$$I_3'(v) \leq \int_{v+\alpha_{v2}z_2\geq v^*} (K_q-1)\nu_2(dz_2) \leq (K_q-1)\nu_2(\mathbb{R}).$$

Therefore, by the condition (3.4), for all $v \geq v^*$

$$\begin{aligned} G'(v) &\leq -\delta - \gamma\mu_c + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + (K_q-1)\nu_2(\mathbb{R}) \\ &\quad + \int_{\mathbb{R}} \left\{K_q(1+\alpha_c z_1)^{-\gamma} - 1 + \gamma\alpha_c z_1\right\}\nu_1(dz_1) \;<\; 0. \end{aligned}$$

Since $G(v^*) = 0$ by construction, it follows that $G(v) \leq 0$ for $v \geq v^*$.

B.2 Condition (ii)

By construction, we have for $v \geq v^*$

$$-\psi_v'(v) + 1 = 0.$$

On the other hand, by Lemma 1, we have for $0 < v < v^*$

$$-\psi_v'(v) + 1 = -\frac{\Psi'(v)}{\Psi'(v^*)} + 1 \leq 0.$$

B.3 Condition (iii)

Recall that

$$\begin{aligned} |\sigma^\top(Y_t^D)\nabla\phi(Y_t^D)|^2 &= \Big|-\gamma\sigma_c e^{-\delta(s+t)} C_t^{-\gamma}\psi_v(V_t^D) + \sigma_v\rho e^{-\delta(s+t)} C_t^{-\gamma}\psi_v'(V_t^D) \\ &\qquad + \sigma_v\sqrt{1-\rho^2} e^{-\delta(s+t)} C_t^{-\gamma}\psi_v'(V_t^D)\Big|^2. \end{aligned}$$

Since $\psi_v'(V_t^D) \leq \max\{1, \psi_v'(0)\}$ by Lemma 1 and $\psi_v(V_t^D) \leq \psi_v(V_t^0) \leq V_t^0 + K_3$, we have

$$|\sigma^\top(Y_t^D)\nabla\phi(Y_t^D)|^2 \leq A_1 e^{-2\delta(s+t)} C_t^{-2\gamma} \left(V_t^0 + A_2\right)^2, \tag{B.6}$$

where A_1, A_2 are some positive constants. Here, we define

$$h_m(t) := \mathbb{E}^y \left[e^{-2\delta(s+t)} C_t^{-2\gamma} \left(V_t^0\right)^m\right],$$

where $m = 0, 1, 2$. Applying the Itô formula, we obtain

$$h_0(t) = h_0(0) + H_0 \int_0^t h_0(s)ds,$$

where

$$H_0 := -2\delta - 2\gamma\mu_c + \gamma(2\gamma+1)\sigma_c^2 + \int_{\mathbb{R}} \left\{(1+\alpha_c z_1)^{-2\gamma} - 1 + 2\gamma\alpha_c z_1\right\} \nu_1(dz_1).$$

Therefore, we obtain

$$h_0(t) = h_0(0)e^{H_0 t}, \qquad \text{for all} \quad t \geq 0.$$

Since $H_0 < 0$ by the assumption (2.4), we have

$$\int_0^\infty h_0(t)dt < \infty. \tag{B.7}$$

Next, we obtain by the Itô formula

$$h_1(t) = h_1(0) + H_0 \int_0^t h_1(s)ds + H_1 \int_0^t h_0(s)ds,$$

where

$$H_1 := \mu_v - 2\gamma\sigma_c\sigma_v\rho + \int_{\mathbb{R}} \left\{(1+\alpha_c z_1)^{-2\gamma}\alpha_{v1}z_1 - \alpha_{v1}z_1\right\} \nu_1(dz_1).$$

Because of (B.7), there exits some constant K_1 such that

$$h_1(t) \leq K_1 + H_0 \int_0^t h_1(s)ds.$$

By Gronwall's inequality, we have $h_1(t) \leq K_1 e^{H_0 t}$. Therefore,

$$\int_0^\infty h_1(t)dt \leq K_1 \int_0^\infty e^{H_0 t} ds < \infty. \tag{B.8}$$

Similarly, we obtain by the Itô formula

$$h_2(t) = h_2(0) + H_0 \int_0^t h_2(s)ds + 2H_1 \int_0^t h_1(s)ds + H_2 \int_0^t h_0(s)ds,$$

where

$$H_2 := \sigma_v^2 + \int_{\mathbb{R}} (1+\alpha_c z_1)^{-2\gamma} \alpha_{v1}^2 z_1^2 \nu_1(dz_1) + \int_{\mathbb{R}} \alpha_{v2}^2 z_2^2 \nu_2(dz_2).$$

Because of (B.7) and (B.8), there exits some constant K_2 such that

$$h_2(t) \le K_2 + H_0 \int_0^t h_2(s)ds.$$

By Gronwall's inequality, we have $h_2(t) \le K_2 e^{H_0 t}$. Therefore,

$$\int_0^\infty h_2(t)dt \le K_2 \int_0^\infty e^{H_0 t} ds < \infty. \tag{B.9}$$

From (B.6)-(B.9), we conclude

$$\mathbb{E}^y \left[\int_0^\tau |\sigma^\top(Y_t^D)\nabla\phi(Y_t^D)|^2 dt \right] \le A_3 \int_0^\infty \sum_{m=0}^{2} h_m(t)dt < \infty,$$

where A_3 is a positive constant.

Note that

$$\begin{aligned}
&\left|\ \phi(s+t, C_t + \alpha_c C_t z_1, V_t^D + \alpha_{v1} z_1) - \phi(s+t, C_t, V_t^D)\ \right|^2 \\
&= \left|\ e^{-\delta(s+t)} C_t^{-\gamma}(1+\alpha_c z_1)^{-\gamma}\psi_v(V_t^D + \alpha_{v1} z_1) - e^{-\delta(s+t)} C_t^{-\gamma}\psi_v(V_t^D)\ \right|^2 \\
&\le\ e^{-2\delta(s+t)} C_t^{-2\gamma} \left\{ (1+\alpha_c z_1)^{-\gamma} |\psi_v(V_t^D + \alpha_{v1} z_1)| + |\psi_v(V_t^D)| \right\}^2 \\
&\le\ e^{-2\delta(s+t)} C_t^{-2\gamma} \left(V_t^0 + A_4 \right)^2 (1+\alpha_c z_1)^{-\gamma} \quad \text{a.a. } \nu_1,
\end{aligned}$$

where A_4 is a positive constant. Thus, we have

$$\int_{\mathbb{R}} \left|\ \phi(s+t, C_t + \alpha_c C_t z_1, V_t^D + \alpha_{v1} z_1) - \phi(s+t, C_t, V_t^D)\ \right|^2 \nu_1(dz_1) \tag{B.10}$$

$$\le\ A_5 e^{-2\delta(s+t)} C_t^{-2\gamma} \left(V_t^0 + A_4 \right)^2, \tag{B.11}$$

where A_5 is a positive constant. Therefore, from (B.10) and (B.7)-(B.9), we conclude

$$\begin{aligned}
&\mathbb{E}^y \left[\int_0^\tau \int_{\mathbb{R}} \left|\ \phi(s+t, C_t + \alpha_c C_t z_1, V_t^D + \alpha_{v1} z_1) - \phi(s+t, C_t, V_t^D)\ \right|^2 \nu_1(dz_1)dt \right] \\
&\le\ A_6 \int_0^\infty \sum_{m=0}^{2} h_m(t)dt < \infty,
\end{aligned}$$

where A_6 is a positive constant. In a similar manner, we obtain

$$\mathbb{E}^y \left[\int_0^\tau \int_{\mathbb{R}} \left|\ \phi(s+t, C_t, V_t^D + \alpha_{v2} z_2) - \phi(s+t, C_t, V_t^D)\ \right|^2 \nu_2(dz_2)dt \right] < \infty.$$

B.4 Conditions (vii), (viii), and (ix)

Recall that $(V_t^0)_{t\ge 0}$ is a spectrally negative Lévy process that has no positive jumps. Therefore, the dividend process $(D_t^*)_{t\ge 0}$ defined as

$$D_t^* = \max\left(0, \sup_{0\le s\le t} (V_s^0 - v^*)\right),$$

is continuous. Because $dD_t^* = 0$ for $0 < V_t^{D^*} < v^*$ and $-\psi_v'(V_t^{D^*}) + 1 = 0$ for $V_t^{D^*} \ge v^*$, the condition (viii) is satisfied. Since $(D_t^*)_{t\ge 0}$ is continuous, the condition (ix) is obviously satisfied. By the definition of $(D_t^*)_{t\ge 0}$, $dD_t^* = 0$ when $0 \le V_{t-}^{D^*} + dV_t^0 \le v^*$, while $dD_t^* = dV_t^0$ when $V_{t-}^{D^*} + dV_t^0 > v^*$. Then the condition (vii) is also satisfied.

B.5 Condition (x)

By construction, we immediately have

$$\mathbb{E}^y \left[\phi(Y_\tau^{D^*}) \right] = 0.$$

On the other hand, it satisfies that

$$0 \le \mathbb{E}^y \left[\phi(Y_T^{D^*}) \right] \le e^{-\delta(s+T)} \psi_v(v^*) \mathbb{E}^y \left[C_T^{-\gamma} \right] = e^{-\delta s} c^{-\gamma} \psi_v(v^*) e^{BT},$$

where

$$B := -\delta - \gamma \mu_c + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + \int_{\mathbb{R}} \left\{ (1+\alpha_c z_1)^{-\gamma} - 1 + \gamma \alpha_c z_1 \right\} \nu_1(dz_1).$$

Since $B < 0$ by the assumption (2.5), we have

$$\lim_{T\to\infty} \mathbb{E}^y \left[\phi(Y_T^{D^*}) \right] = 0.$$

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