WORKING PAPER SERIES

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October 2, 2018

No. １９８

The Research Institute for Innovation Management, HOSEI UNIVERSITY

Travel time models for the Rack-moving Mobile Robot System

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Abstract: The rack-moving mobile robot (RMMR) system is a special parts-to-picker automated warehousing system that uses hundreds of rack-moving machines to accomplish the repetitive tasks of storing and retrieving parts by lifting and transporting unit racks autonomously. This paper investigates the operation cycle of the rack-moving machine for storage and retrieval from the perspective of the lane depth, especially exploring the particularity of the RMMR system in multi-deep lanes, and proposes expected travel time models of the rack-moving machine for single- and multi-deep layouts of the RMMR system. To validate the effectiveness of the proposed models, an experimental simulation was conducted with a 1–4-deep layout under six scenarios of different numbers of aisles and layers, and results were compared with results obtained using proposed models. The paper presents useful guidelines for the configuration of the RMMR system layout including the determination of the optimal lane depth.

Keywords: rack-moving mobile robot system; travel time model; lane depth; simulation; warehousing

1 Introduction

The picking pattern adopted in warehouses has gone through three stages with the continuous development of new technologies. *Picker-to-parts* was the pattern adopted in the first stage and was widely applied in warehouses. According to this pattern, human pickers move to shelves to pick orders with or without forklifts. In the second stage, multi-layer shelves were introduced to improve space utilization for effective storage, and block-stacking storage racks were widely applied (Berry 1968; Marsh 1979; Goetschalckx 1987) and automated storage and retrieval systems (AS/RSs) were introduced in the 1950s and first installed in the 1960s to improve picking efficiency and space utilization. The picking pattern adopted in the second stage is *parts-to-picker*, whereby pickers are located at fixed picking positions and a combination of a stack crane and conveyors is used to transport goods to the pickers. Many types of AS/RSs have been widely researched; e.g., unit-load, miniload, autonomous vehicle or shuttle-based, flow-rack, three-dimensional (3D) compact, and vertical lift module AS/RSs. The application of an automated guide vehicle and the

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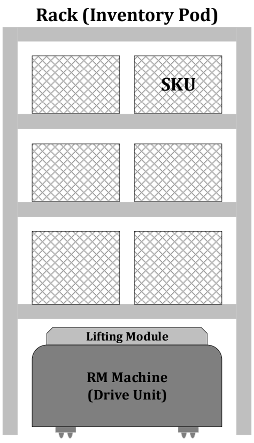
rack-moving mobile robot (RMMR) system marked the arrival of the third stage (Boysen et al. 2017).

The RMMR system is a specific parts-to-picker system, in which a mobile robot, called the rack-moving (RM) machine, lifts a rack and transports it to a stationary picker, as shown in Figure 1. In this picking pattern, the drive unit machine picks up not only goods but also racks, which implies the racks are transportable in the warehouse.

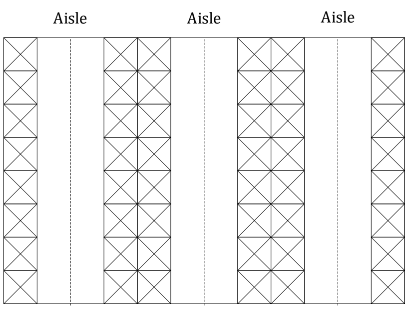


(a) Movable rack and RM machine

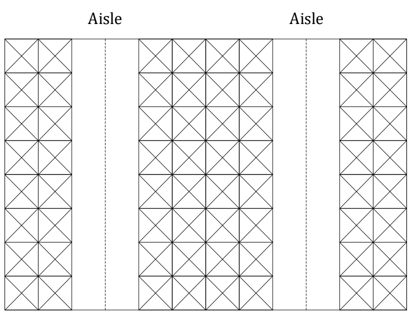
(b) Kiva system (Kirks et al. 2012)

Fig. 1. RMMR system

A well-known RMMR system is the Kiva system, which was first installed and deployed in 2006. The Kiva system has several advantages over the AS/RS, such as greater accountability, no downstream dependency, no batch processing, location-free replenishment, adaptive slotting, no single point of failure, rapid deployment, spatial flexibility, and expandability (Wurman et al. 2008). The function of the RM machine is similar to that of cranes and shuttles in the AS/RS. The RM machine moves from the dwell point to the target rack in any lane position of the storage area and then lifts the target rack and transports it to the pickup and delivery (P/D) station (called the workstation in this paper). This differs from the function of cranes in an AS/RS, The basic function of the RM machine is thus the lifting and transportation of a rack.



(a) Single-deep layout



(b) Double-deep layout

Fig. 2. Rack layout in the RMMR system

Figure 2 shows two typical rack layouts for the RMMR system. The flexibility of the racks and RM machine (Wurman et al. 2008; Boysen et al. 2017) means that there are several differences from the lane depth problem of the AS/RS; e.g., (1) hundreds of RM machines work at the same time, in contrast to the combined operations of cranes and shuttles, (2) the RM machine can move in any horizontal or vertical direction, even below the racks, in contrast to the aisle-captive or shuttle-based AS/RS, (3) the rack layout has a certain flexibility with lower setup costs and can have multiple deep lanes for rack placement, (4) the dwell point of the RM machine is an arbitrary position, depending on the previous operation order, in contrast to the fixed position for stacker cranes, and (5) the utilization of the racks and storage area is higher and the probability of multiple movements of racks is higher because of the shared storage policy. Therefore, the operation cycle of the RM machine differing from that of stacker cranes needs to be considered in depth for new travel characteristics.

This paper considers a special parts-to-picker system, which is called the rack-moving mobile robot system and well known as the Kiva system. The aim of the paper is to illustrate the operation cycle of the RM machine for storage and retrieval from the perspective of lane depth, and especially to explore the particularity of the RMMR system having a multi-deep layout and to propose expected travel time models of the RM machine for single- and multiple-deep layouts in the RMMR system.

The remainder of the paper is organized as follows. Section 2 presents a literature review of RMMR systems and discusses related work on AS/RSs. Section 3 illustrates the generic layout of the RMMR system and the operational command cycle for different lane depths. Section 4 presents an analytical travel model for the single-deep layout, while Section 5 does the same for the multi-deep layout. Section 6 presents a simulation implementation and compares the results obtained by simulation with those obtained using proposed models. Finally, Section 7 presents conclusions and directions of further research.

2 Literature review

Since a patent of a material handling system went to Mountz in 2005 and a method employing mobile autonomous inventory trays and peer-to-peer communications was proposed, a series of rack-moving mobile robot systems has been installed and operated; e.g., Kiva, KARIS, and ADAM (Kirks et al. 2012). Guizzo (2008) and Wurman et al. (2008) introduced the configuration, operation procedure, and technology of the Kiva system. Boysen et al. (2017) focused on order processing in an RMMR system and proposed a dynamic algorithm to solve the tradeoff problem considering the sequencing of picking orders and the sequencing of arrival racks. Other scholars focused on coordinating hundreds of autonomous vehicles. Herrero-Pérez and Martínez-Barberá (2011) presented the development of a distributed transportation system composed of a team of flexible automated guide vehicles. Yu (2016) analyzed the intractability of optimal multirobot path planning on planar graphs. To the best of our knowledge, the study of Lamballais (2017) was unique in focusing on the travel time of the RM machine. Lamballais developed queueing network models for single-line and multi-line orders to analytically estimate the maximum order throughput, average order cycle time, and robot utilization.

Although there has been no previous travel time analysis from the perspective of the lane depth for RMMR systems, relevant historical studies on lane depth have been conducted for the block stacking storage system, unit-load AS/RS, and AVS/RS.

Concerning the lane depth of a block stacking storage system, Kind (1965) proposed a formula for the best unique lane depth for a single product with respect to space utilization. Mastson and White (1981) estimated the lane depth and proposed an approximate formula with small relative error. Kooy (1975) analyzed the tradeoff between inbound and outbound flows and between required and available lanes, and Kooy (1981) then obtained optimal multiple lane depths for a product through complete enumeration over all feasible combinations of lane depths. Matson and White (1981, 1984) presented a formula for the optimal continuous single-lane depth with the objective of space utilization. Considering the dynamic characteristics of a deep lane, Goetschalckx (1987) presented an algorithm to compute the optimal number of lanes and optimal lane depths for a single product, when the lanes are allowed to have different depths. Meanwhile, he analyzed the lane depths for multiple products and shared storage policies. Larson et al. (1997) proposed a heuristic approach to obtain the optimal layout of a block stacking warehouse with the double objective of maximizing space utilization and minimizing transportation costs. Derhami et al. (2017) proposed mathematical models to obtain the optimal lane depth for single and multiple stock keeping units under the constraint of a finite production rate and used a simulation model to evaluate the performance of the proposed models. In a book on warehouse and distribution science, Bartholdi and Hackman (2017) developed Matson's model to optimize volume utilization instead of floor utilization and computed the optimal lane depth for the rack layout configuration with four pallets of the stock keeping unit.

AS/RSs have been widely applied over decades for intelligent manufacturing and e-commerce retail. Various reviews on warehouse design and control, especially on the AS/RS, have been presented in recent years. De Koster et al. (2007) and Gu et al. (2010) focused on the design and performance of different types of warehouse while Roodbergen and Vis (2009), Gagliardi et al. (2012), Vasili et al. (2012), and Boysen and Stephan (2016) focused on the design and planning models of the AS/RS. Most research has focused on the travel time of the stacker crane, which is a pickup and delivery machine, for different influencing factors and environments; e.g., different types of AS/RS, speed profiles of the crane, storage assignment policies, command cycles, rack depths, and sequencing of requests of the crane.

For the generic unit-load AS/RS, most studies have considered single-deep or double-deep lanes, with the main objective being the travel time of the stacker crane. Although various studies have focused on the background of the single-deep system, the double-deep system is also gaining attention. The advantage of the double-deep storage rack is that fewer aisles are needed, resulting in a more efficient use of floor space. Many scholars have focused on the double-deep system. Oser and Ritonja (2004) proposed travel time models for two scenarios of single- and double-deep systems in the case of a class-based storage policy. Lerher (2010) proposed new analytical travel time models that can be used to calculate the mean cycle time for single- and dual-command cycles in a double-deep AS/RS.

Another typical automated storage and retrieval system is the AVS/RS, which is also called shuttle-based storage and retrieval system (SBS/RS). Since the SBS/RS was developed by Carlo and Vis in 2012, the performance of the AVS/RS has been investigated through simulation (Smew et al. 2013), experimental validation (Sari et al. 2014), and computational analysis (Lerher et al. 2015). However, all previous research was based on a single-deep system until Lerher (2016) firstly developed a travel time model for the double-deep SBS/RS. Lerher proposed a travel time model considering the real operating characteristics of the elevator lifting table and the shuttle carrier under the condition that blocking totes are repositioned to the nearest free storage location during the retrieval of the shuttle carrier. Furthermore, on the basis of the research of Fukunari and Malmorg (2008), Manzini (2016) proposed a set of original analytic models to determine the number and depth of lanes of the AVS/RS using lane-depth storage.

Other AS/RSs, such as the flow-rack AS/RS with gravity racks (Sari et al. 2005; Chen et al. 2015) and 3D compact AS/RS with a conveyor in the depth direction (De Koster et al. 2006; Yu and De Koster 2009a, 2009b; Yang et al. 2017), are considered as multiple-deep AS/RSs. However, the configuration of racks and operation procedure of the P/D machine are different from those of the generic block-stacking system and AVS/RS.

According to the above review of the literature, space utilization is usually regarded as an important objective of performance from the perspective of the lane depth of block stacking storage systems; nevertheless, travel time models have been developed for various types of AS/RS with a certain lane depth, most being single or double deep. In studies of the RMMR system, there has been little investigation of the travel time of the RM machine, especially from the perspective of the lane depth. In contrast to existing studies, we focus on the operation cycle of the RM machine in single- and multiple-deep systems and explore travel time models to estimate the performance of the RMMR system.

3 RMMR system

This section illustrates observed system configurations, including the layout of the RMMR system and the operation sequence of the RM machine in a cycle.

3.1 RMMR system layout

In proposing a travel time model, the following basic assumptions describing the RMMR system are considered for characteristics of the layout.

H1. A shared assignment policy is considered, which means that any position of racks is equally likely to be selected for storage, and each rack holds unfixed types of items and goods.

H2. RM machines are allowed to travel in horizontal and vertical directions independently even under racks with no load.

H3. There are sufficient RM machines to support all required operations of retrieval and refilling simultaneously.

H4. The dwell point of the RM machine is located where the previous one operation cycle was completed.

H5. The assignment of the RM machine followed a randomized policy, which means RM machines can serve arbitrary racks across aisles and depth.

The following notations and symbols (see Figure 3) are used in the paper.

n number of vertical aisles in the storage area

p number of layers

d lane depth in the storage area

v velocity of the RM machine

L_l length of a rack position

W_l width of a rack position

W_a width of an aisle

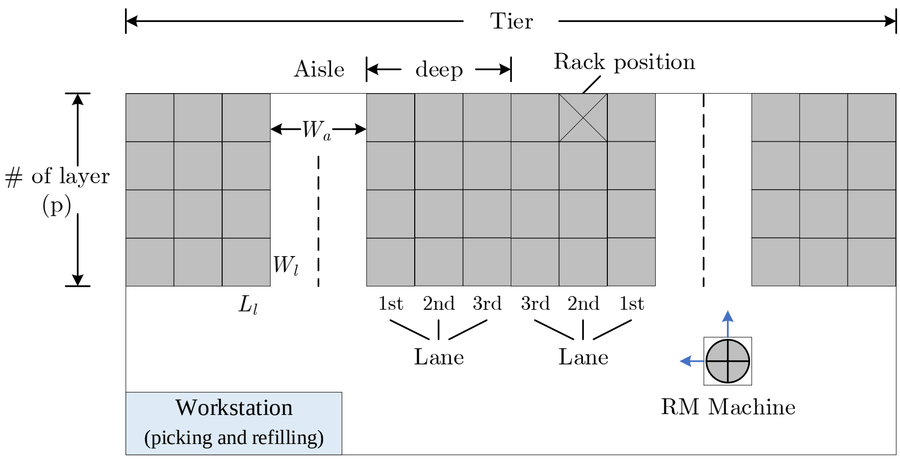


Fig. 3. Generic layout of the RMMR system

In Figure 3, a three-deep RMMR system is constructed to illustrate a multiple-deep RMMR system. In each layer, there are three rack positions on each side of a picking aisle, and racks are located at one position, which is defined by an ordinal number as the first, second, or third rack position. The picking and refilling operations are conducted at the same location of the workstation.

3.2 Operation cycle of the RM machine in the RMMR system

On the basis of Figure 3 and the general assumptions described above, the operation cycle of the RM machine in the single-deep RMMR system can be divided into three generic parts, namely moving from the dwell point to the target rack directly, lifting and transporting the target rack to the workstation along aisles to pick and refill, and transporting the rack back to the primary position after refilling. There is an additional operation of rack arrangement in the multiple-deep RMMR system for moving a deeper rack to the aisle.

3.2.1 Operation cycle of the RM machine in a single-deep RMMR system

According to assumption H4, we suppose that the RM machine stays at the initial dwell point (denoted O_r). The sequence of operations (see the orange dashed lines in Fig. 4) is as follows.

(1) The RM machine moves to the target rack position (denoted Lt_1) along horizontal and vertical lines when the order-retrieval command is given.

(2) The RM machine lifts the target rack and transports it to the workstation along aisles in horizontal and vertical directions. The operations of picking and refilling are carried out at the workstation (denoted by R_1).

(3) The RM machine with the refilled rack returns to the primary position Lt_1, which is the dwell point for the next operation cycle (see the green dashed lines in Fig. 4).

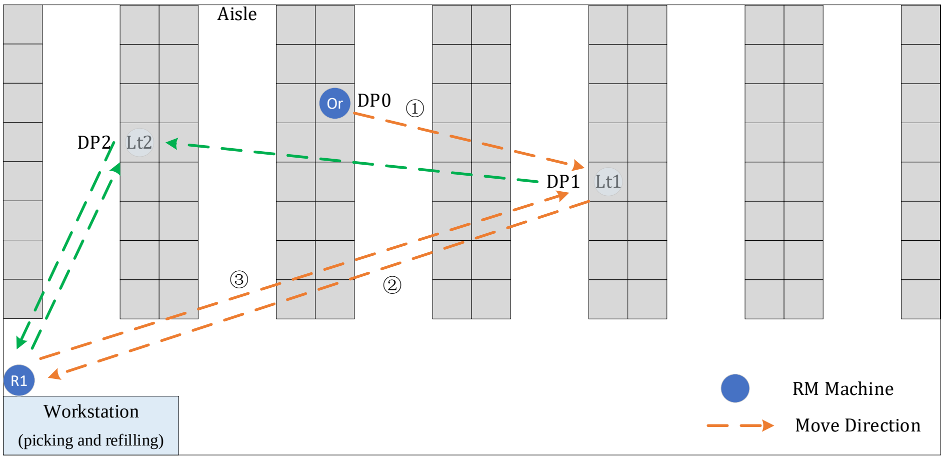


Fig. 4. Command cycle of an RM machine in a single-deep RMMR system

When developing the proposed analytical travel time models for the RMMR system, the following basic simplifications and assumptions are made.

H6. The time required by the RM machine to lift and put down of racks and the time spent waiting to pick and refill are ignored.

H7. The congestion and avoidance of RM machines is not considered.

H8. The travel time between two arbitrary positions in the RMMR system is symmetrical and does not change over time.

H9. The acceleration and deceleration of the RM machine are assumed instantaneous and ignored.

Additional notations are used in proposing the travel time model for the single-deep

RMMR system:

O_1 position of the workstation in the RMMR system,

O_r position of the dwell point of the RM machine in one cycle,

L_t position of the target rack,

$T_s^{(1)}$ expected travel time of the RM machine between two arbitrary positions in the single-deep system, and

$T_{RMM}^{(1)}$ expected travel time of the RM machine in one cycle of the single-deep system.

Let $T_s^{(1)}(O_r, L_t)$ be the expected travel time from initial dwell point O_r to lane position L_t of the target rack. Let $T_s^{(1)}(O_1, L_t)$, which is equal to $T_s^{(1)}(L_t, O_1)$, be the expected travel time from workstation O_1 to lane position L_t of the target rack.

Therefore, according to assumptions H6–H10, the expected travel time of the RM machine in a cycle of the single-deep RMMR system can be represented as

$$T_{RMM}^{(1)} = T_s^{(1)}(O_r, L_t) + 2T_s^{(1)}(O_1, L_t). \tag{1}$$

3.2.2 Operation cycle of an RM machine in a multi-deep RMMR system

The operation cycle of a single-deep RMMR system also applies to the multi-deep system. In contrast to the case of the single-deep RMMR system, however, two new characteristics need to be considered.

(1) An additional rack arrangement needs to be considered because the target rack cannot be transported outside when it is located in a deep lane, which implies that the rearrangement strategy and procedure need to be set and the rearrangement time (denoted T_f) needs to be calculated.

(2) Because the aim of the arrangement is to move the target rack to the outside, when the RM machine turns back from the workstation, only the outside lane position is free. Therefore, the dwell point of the RM machine is always located in the first lane position of any layer in the multi-deep RMMR system (see Fig. 5).

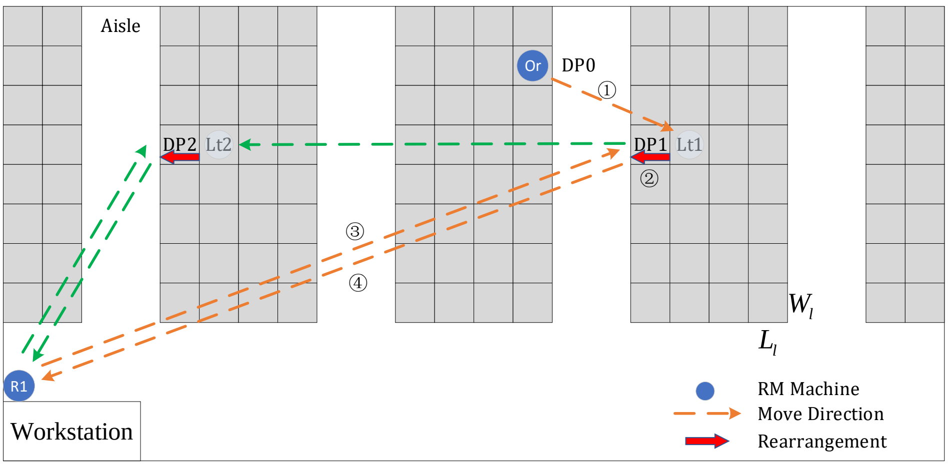


Fig. 5. Command cycle of the RM machine in the multi-deep RMMR system

Additional notations used in proposing the travel time model in the multi-deep RMMR system are

L_o outmost position of the lane where the target rack is located,

$T_s^{(d)}$ expected travel time of the RM machine between two arbitrary positions in the multiple-deep system,

$T_{RMM}^{(d)}$ expected travel time of the RM machine in one cycle of the multiple-deep system, and

T_f expected rearrangement time for the rack in the deep lane.

In the multi-deep RMMR system, let $T_s^{(d)}(O_r, L_t)$ be the expected travel time from initial dwell point O_r to lane position L_t of the target rack; let $T_s^{(d)}(O_1, L_o)$, which is equal to $T_s^{(d)}(L_o, O_1)$, be the expected travel time from the workstation O_1 to outmost lane L_o of the target rack location.

The expected travel time of the RM machine in a cycle of the multi-deep RMMR system is therefore

$$T_{RMM}^{(d)} = T_s^{(d)}(O_r, L_t) + 2T_s^{(d)}(O_1, L_o) + T_f. \tag{2}$$

4 Expected travel time model for the single-deep RMMR system

The analytical travel time model for the single-deep RMMR system is first presented in detail. According to equation (1), the terms $T_s^{(1)}(O_1, L_t)$ and $T_s^{(1)}(O_r, L_t)$ need to be derived firstly, and the expected travel time for the single-deep RMMR system can then be calculated by summing $T_s^{(1)}(O_1, L_t)$ and $T_s^{(1)}(O_r, L_t)$.

4.1 Derivation of $T_s(O_1, L_t)$

Let $T_x^{(1)}$ be the expected horizontal travel time between any two lanes and $T_y^{(1)}$ be the expected vertical travel time between any two lanes. The expected travel time from workstation O_1 to lane position L_t of the target rack, denoted $T_s^{(1)}(O_1, L_t)$, is the sum of travel times in the horizontal and vertical directions, respectively denoted $T_x^{(1)}(O_1, L_t)$ and $T_y^{(1)}(O_1, L_t)$.

According to the randomized storage policy, there is equal probability of the target rack position being located in each tier. Let x_{ij} be horizontal coordinate of the lane for the ith tier and jth layer. $T_x^{(1)}(O_1, L_t)$ can then be derived as

$$
\begin{aligned}
T_x^{(1)}(O_1,L_t) &= \frac{1}{v}\cdot\frac{1}{2n}\cdot\left[\sum_{i=1}^{2n} x_{ij} + n(W_l+W_a)\right] \\
&= \frac{1}{2nv}\cdot\left[\begin{array}{l}\frac{1}{2}W_l+\left(\frac{3}{2}W_l+W_a\right)+\left(\frac{5}{2}W_l+W_a\right)+\left(\frac{7}{2}W_l+2W_a\right)+\cdots \\ +\left(\left(\frac{4n-3}{2}\right)W_l+(n-1)W_a\right)+\left(\left(\frac{4n-1}{2}\right)W_l+nW_a\right)+n(W_l+W_a)\end{array}\right]. \\
&= \frac{2n+1}{2v}W_l+\frac{n+1}{2v}W_a
\end{aligned}
\tag{3}
$$

Likewise, let y_{ij} be the vertical coordinate of the ith tier and jth layer. $T_y^{(1)}(O_1,L_t)$ can then be derived as

$$
\begin{aligned}
T_y^{(1)}(O_1,L_t) &= \frac{1}{v}\cdot\frac{1}{p}\cdot\sum_{j=1}^{p} y_{ij} \\
&= \frac{1}{v}\cdot\frac{1}{p}\left[\left(W_a+\frac{1}{2}L_l\right)+\left(W_a+\frac{3}{2}L_l\right)+\left(W_a+\frac{5}{2}L_l\right)+\cdots+\left(W_a+\frac{2p-1}{2}L_l\right)\right]. \\
&= \frac{1}{v}W_a+\frac{p}{2v}L_l
\end{aligned}
\tag{4}
$$

Then,

$$
T_s^{(1)}(O_1,L_t) = T_x^{(1)}(O_1,L_t) + T_y^{(1)}(O_1,L_t) = \frac{1}{2v}\left[(n+3)W_a+(2n+1)W_l+pL_l\right]. \tag{5}
$$

4.2 Derivation of $T_s(O_r,L_t)$

The term $T_s^{(1)}(O_r,L_t)$ is the sum of $T_x^{(1)}(O_r,L_t)$ and $T_y^{(1)}(O_r,L_t)$, which are the horizontal and vertical expected travel times respectively:

$$
T_s^{(1)}(O_r,L_t) = T_x^{(1)}(O_r,L_t) + T_y^{(1)}(O_r,L_t). \tag{6}
$$

First, according to assumption H8, there is equal probability of the dwell point O_r being located in any lane position. For the target position, the expected travel time of the RM machine depends on which tier the dwell point is located. Let i_{Or} denote the index of racks on the tier, starting from the coordinate origin and increasing in the positive direction, and let $T_{x,i_{or}}^{(1)}(O_r,L_t)$ denote the value of $T_x^{(1)}(O_r,L_t)$ when the dwell point of the RM machine is located in the i_{Or}th column.

When $i_{Or}=1$, according to assumption H6, $T_{x1}^{(1)}(O_r,L_t)$ can be derived as

$$
\begin{aligned}
T_{x1}^{(1)}(O_r,L_t) &= \frac{1}{v}\cdot\frac{1}{2n}\cdot\left[\sum_{i=1}^{2n}(x_{ij}-x_{1j})\right] \\
&= \frac{1}{v}\cdot\frac{1}{2n}\cdot\left[\begin{array}{l}0+(W_l+W_a)+(2W_l+W_a)+(3W_l+2W_a)+\cdots \\ +((2n-2)W_l+(n-1)W_a)+((2n-1)W_l+nW_a)\end{array}\right]. \\
&= \frac{1}{2vn}\cdot\left[(2n^2-n)W_l+n^2W_a\right]
\end{aligned}
\tag{7}
$$

Likewise, $T_{x,i_{or}}^{(1)}(O_r,L_t)$ for different values of i_{Or} can be expressed as

$$
T_{x2}^{(1)}(O_r,L_t) = \frac{1}{2vn}\left[(2n^2-3n+2)W_l+(n^2-2n+2)W_a\right], \tag{8}
$$

$$T_{x3}^{(1)}\ O_r, L_t\ = \frac{1}{2vn}\left[\ 2n^2 - 5n + 6\ W_l +\ n^2 - 2n + 2\ W_a\right], \tag{9}$$

$$T_{x4}^{(1)}\ O_r, L_t\ = \frac{1}{2vn}\left[\ 2n^2 - 7n + 12\ W_l +\ n^2 - 4n + 8\ W_a\right], \tag{10}$$

$$T_{x,2n}^{(1)}\ O_r, L_t\ = \frac{1}{2vn}\left[2n^2 -\ 4n - 1\ n + 2n\ 2n - 1\ W_l + \left(n^2 -\ 2n\ n + 2 \cdot \left(\frac{2n}{2}\right)^2\right)W_a\right]. \tag{11}$$

$T_x^{(1)}\ O_r, L_t$ is therefore the mean of $T_{x,i_{or}}^{(1)}\ O_r, L_t$ for all values of i_{Or}. The expression is

$$\begin{aligned} T_x^{(1)}\ O_r, L_t\ &= \frac{1}{2n}\sum_{i_{Or}} T_{x,i_{or}}^{(1)}\ O_r, L_t \\ &= \frac{1}{4vn^2}\left[\begin{array}{c}\left(2n \cdot 2n^2 - 4n^2 \cdot n + \frac{2n^{\ 3} - 2n}{3}\right)W_l + \\ \left(2n \cdot n^2 - \left(\frac{4n^2}{2}\right) \cdot n + \frac{2n^{\ 3} + 4n}{6}\right)W_a\end{array}\right]. \\ &= \frac{1}{6nv}\left[\ 4n^2 - 1\ W_l +\ 2n^2 + 1\ W_a\right] \end{aligned} \tag{12}$$

Second, for the expected travel time in the vertical direction, let $T_{y,i_{or}}^{(1)}\ O_r, L_t$ denote the value of $T_y^{(1)}\ O_r, L_t$ when the dwell point of the RM machine is located in the j_{Or} th row. Similarly, $T_{y,i_{or}}^{(1)}\ O_r, L_t$ can be derived through the analysis of different values of j_{Or} :

$$T_{y1}^{(1)}\ O_r, L_t\ = \frac{1}{v} \cdot \frac{1}{p} \cdot \left[\sum_{j=1}^{p}\ y_{ij} - y_{i1}\ \right] = \frac{1}{pv}\left[0 + \frac{p\ \ p-1}{2}\right]L_l = \frac{1}{pv}\left(\frac{p^2 - p}{2}\right)L_l, \tag{13}$$

$$T_{y2}^{(1)}\ O_r, L_t\ = \frac{1}{pv}\left(2 + \frac{p^3 - 3p}{2}\right)L_l, \tag{14}$$

$$T_{yp}^{(1)}\ O_r, L_t\ = \frac{1}{pv}\left(p\ \ p - 1\ + \frac{p^2 -\ 2p - 1\ p}{2}\right)L_l. \tag{15}$$

$T_y^{(1)}\ O_r, L_t$ is therefore the mean of $T_{y,i_{or}}^{(1)}\ O_r, L_t$ for all values of i_{Or}. The expression is

$$T_y^{(1)}\ O_r, L_t\ = \frac{1}{p}\sum_{j_{Or}} T_{y,i_{or}}^{(1)}\ O_r, L_t\ = \frac{1}{p} \cdot \frac{1}{pv}\left[\frac{p^3 - p}{3} + p \cdot \frac{p^2}{2} - \left(\frac{p^2}{2}\right) \cdot p\right]L_l = \frac{p^2 - 1}{3vp}L_l. \tag{16}$$

So far, according to equations (6), (12), and (16), the expected travel time from dwell point O_r to lane position L_t of the target rack can be expressed as

$$T_s^{(1)}\ O_r, L_t\ = \frac{1}{6nv}\left[\ 4n^2 - 1\ W_l +\ 2n^2 + 1\ W_a\right] + \frac{p^2 - 1}{3vp}L_l. \tag{17}$$

4.3 Equation of the expected travel time in a single deep environment

We aggregate equations (5) and (17) to calculate $T_{RMM}^{(1)}$. The expected travel time of the RM machine for complete operation in a single-deep environment can then be expressed as

$$T_{RMM}^{(1)} = \frac{1}{3v}\left(\frac{8n^2+18n+1}{2n}W_a + \frac{16n^2+6n-1}{2n}W_l + \frac{4p^2-1}{p}L_l\right). \tag{18}$$

5 Expected travel time model for a multiple-deep RMMR system

On the basis of the expected travel time model in the single-deep environment of the RMMR system, we turn the discussion to the expected travel time model in the multiple-deep environment. According to equation (2), the terms $T_s^{(d)}(O_1,L_o)$, $T_s^{(d)}(O_r,L_t)$, and T_f need to be derived firstly, and the expected travel time for the multi-deep RMMR system can then be calculated as the sum of $T_s^{(d)}(O_1,L_o)$, $T_s^{(d)}(O_r,L_t)$, and T_f.

5.1 Derivation of $T_s(O_r,L_t)$

Similar to the analysis of the derivation in section 4.2, the expected travel time $T_s^{(d)}(O_r,L_t)$ can be presented as the sum of the horizontal travel time (denoted $T_x^{(d)}(O_r,L_t)$) and vertical travel time (denoted $T_y^{(d)}(O_r,L_t)$):

$$T_s^{(d)}(O_r,L_t) = T_x^{(d)}(O_r,L_t) + T_y^{(d)}(O_r,L_t). \tag{19}$$

First, the expected travel time in the vertical direction is similar to that in the scenario of the single-deep system, and the equation is the same as equation (16):

$$T_y^{(d)}(O_r,L_t) = \frac{p^2-1}{3vp}L_l. \tag{20}$$

Second, the expected travel time in the horizontal direction is similar to that in the scenario of the single-deep system but there is a new factor of the lane depth influencing the travel time. i_{Or} is the index of racks in the column, starting from the coordinate origin and increasing in the positive direction. Let $T_{x,i_{or}}^{(d)}(O_r,L_t)$ denote the value of $T_x^{(d)}(O_r,L_t)$ when the dwell point of the RM machine is located in the i_{Or} th column.

Let x_{ijk} be the horizontal coordinate of the kth position in column i and row j. $T_{x,i_{or}}^{(d)}(O_r,L_t)$ can be derived from the value of i_{Or} in turn from 1 to 2n:

$$\begin{aligned} T_{x1}^{(d)}(O_r,L_t) &= \frac{1}{v}\cdot\frac{1}{2n}\cdot\left[\sum_{i=1}^{2n}(x_{ij1}-x_{1j1})\right] \\ &= \frac{1}{2vn}\left[\begin{aligned}&0+(W_l+W_a)+(2dW_l+W_a)+((2d+1)W_l+2W_a)+\cdots\\&+((2dn-2d)W_l+(n-1)W_a)+((2dn-2d+1)W_l+nW_a)\end{aligned}\right] \\ &= \frac{1}{2vn}\left[((2n^2-2n)d+n)W_l+n^2W_a\right] \end{aligned} \tag{21}$$

Likewise,

$$T_{x2}^{(d)}(O_r,L_t)=\frac{1}{2vn}\Big[\big((2n^2-2n)d+2-n\big)W_l+\big(n^2-2n+2\big)W_a\Big], \tag{22}$$

$$T_{x3}^{(d)}(O_r,L_t)=\frac{1}{2vn}\Big[\big((2n^2-6n+8)d+n-2\big)W_l+\big(n^2-2n+2\big)W_a\Big], \tag{23}$$

$$T_{x4}^{(d)}(O_r,L_t)=\frac{1}{2vn}\Big[\big((2n^2-6n+8)d+4-n\big)W_l+\big(n^2-4n+8\big)W_a\Big], \tag{24}$$

……

$$T_{x,2n}^{(d)}(O_r,L_t)=\frac{1}{2vn}\left[\begin{array}{r}\big((2n^2-2\cdot(2n-1)n+(2n-2)\cdot 2n)d+2n-n\big)W_l\\ +\left(n^2-2n\cdot n+2\cdot\left(\frac{2n}{2}\right)^2\right)W_a\end{array}\right]. \tag{25}$$

The expected travel time $T_x^{(d)}(O_r,L_t)$ is therefore the mean of $T_{x,i_{or}}^{(d)}(O_r,L_t)$ for all values of i_{Or}. The expression is

$$\begin{aligned}T_x^{(d)}(O_r,L_t)&=\frac{1}{2n}\sum_{i_{Or}}T_{x,i_{or}}^{(d)}(O_r,L_t)\\&=\frac{1}{2n}\cdot\frac{1}{3v}\Big[\big(4(n^2-1)d+3\big)W_l+\big(2n^2+1\big)W_a\Big].\\&=\frac{1}{6nv}\Big[\big(4(n^2-1)d+3\big)W_l+\big(2n^2+1\big)W_a\Big]\end{aligned} \tag{26}$$

According to equations (19), (20), and (26), $T_s^{(d)}(O_r,L_t)$ can be written as

$$T_s^{(d)}(O_r,L_t)=\frac{1}{3v}\left[\frac{4(n^2-1)d+3}{2n}W_l+\frac{2n^2+1}{2n}W_a+\frac{p^2-1}{p}L_l\right]. \tag{27}$$

5.2 Derivation of $T_s(O_1,L_o)$

According to the operation procedure of the RM machine in the multi-deep RMMR system, the start position is the first outside lane for the picking operation of the RM machine. The travel time $T_s^{(d)}(O_1,L_o)$ therefore represents the time cost on the route from the first outside lane to the workstation.

In the same way, $T_s^{(d)}(O_1,L_o)$ can be expressed as the sum of the horizontal travel time (denoted $T_x^{(d)}(O_1,L_o)$) and vertical travel time (denoted $T_y^{(d)}(O_1,L_o)$):

$$T_s^{(d)}(O_1,L_o)=T_x^{(d)}(O_1,L_o)+T_y^{(d)}(O_1,L_o). \tag{28}$$

First, because the vertical travel route is not affected by the lane depth, the vertical travel time is the same as that in the single-deep system:

$$T_y^{(d)}(O_1,L_o)=\frac{1}{v}W_a+\frac{p}{2v}L_l. \tag{29}$$

Second, the horizontal travel time is obviously connected with the lane depth, it is necessary to analyze the travel time from the target rack when the lane is in different columns, and the expectation of the horizontal travel time then needs to be calculated assuming that the probability of the lane being in different columns is the same. The analysis procedure is similar to that for the single-deep system and the final expression is

$$T_x^{(d)}\ \ O_1, L_o \ = \frac{2dn+1}{2v} W_l + \frac{n+1}{2v} W_a \,. \tag{30}$$

According to equations (28), (29), and (30), the travel time of the RM machine from the workstation to the target lane is

$$T_x^{(d)}\ \ O_1, L_o \ = \frac{2dn+1}{2v} W_l + \frac{n+3}{2v} W_a + \frac{p}{2v} L_l \,. \tag{31}$$

5.3 Rearrangement strategy and derivation of T_f

Because the inventory can be retrieved in parallel in the Kiva environment (Wurman 2008), the number of RM machines in one aisle is at least one and multiple robots can work simultaneously. For the multi-deep AS/RS, a rearrangement operation is required when the picking position is located in a deeper lane and there are goods located in the front lane. Generally, the rearrangement strategy is that the stacker crane or shuttle is used as temporary storage, ensuring the goods in the deeper lane can be picked. However, the stacker crane or shuttle can only support the temporary storage of one transport unit load (TUL). Lerher (2010) proposed a rearrangement sequence of the TUL for the double-deep unit-load AS/RS. The procedure of rearrangement is that goods in the first lane are picked and placed at the nearest free storage location; the definition of the rearrangement distance is proposed to support rearrangement. This is a common method of rearrangement in double-deep AS/RSs.

Unfortunately, the RMMR system has new characteristics differing from the AS/RS. On the one hand, the lane depth can exceed 2, which implies that two or more racks need to be rearranged at a time, especially for a layout that has a depth greater than 2. Because RM machines can work simultaneously, more robots can move in a certain aisle at the same time, and the routes of other machines can be disturbed if the traditional rearrangement procedure is adopted. On the other hand, considering the flexibility of the motion of the RM machine, rack layout, and space utilization, the proportion of rack utilization is 100% in the storage area, which implies that each storage position has a rack located in the lane.

A new rearrangement strategy is therefore proposed for the RMMR system in this paper. Because the RM machine can move under the racks and each rack can be lifted and transported, the loop rearrangement route is constructed by combining the target lane, arbitrary adjacent lane, and vertical aisle. The procedure is that the racks are transported in the anticlockwise direction by the RM machine in turn beginning with the moving of the front rack, the RM machine then returns to the nearest lane in the clockwise direction, and the procedure is repeated until the target rack is moved to the outside.

The examples of the 2-deep and 3-deep rearrangement procedures are respectively illustrated as Figures 6 and 7.

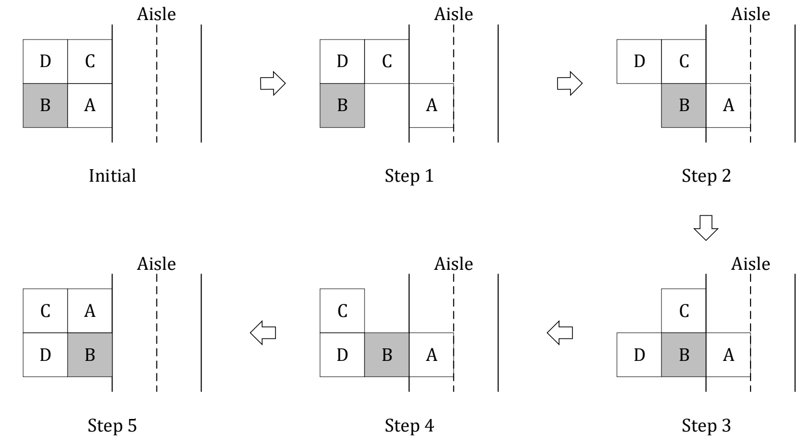


Fig. 6. Procedure of rearranging TULs for double-deep lanes

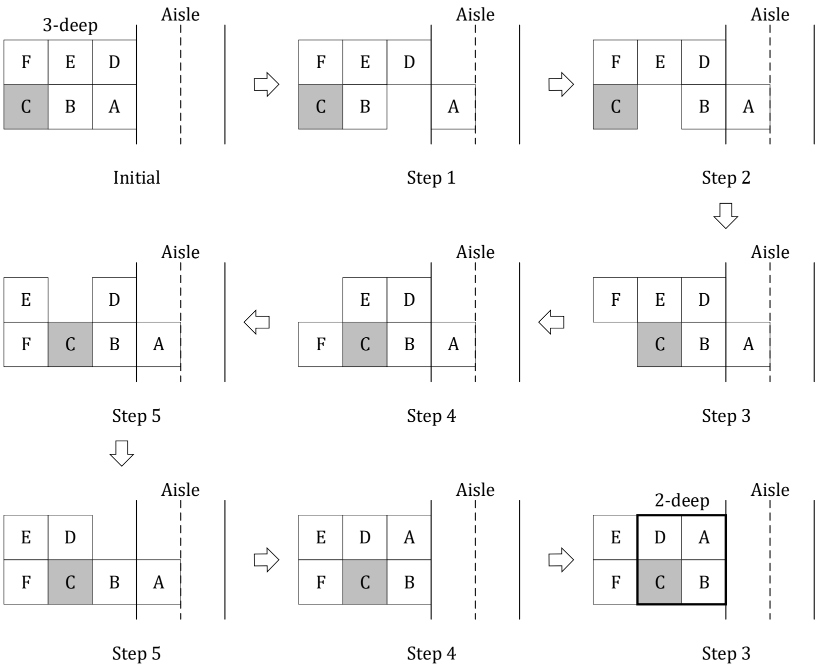


Fig. 7. Procedure of rearranging TULs for 3-deep lanes

As the storage location of the target rack can differ, the number of rack movements in the rearrangement operation can differ, resulting in different rearrangement travel times. It is therefore necessary to analyze the arrangement travel time according to different storage locations of the rack. It is assumed that the probability of the storage location of the target rack is the same in each rack position, and the expected travel time is the mean of all arrangement travel times for the different rack positions. On the basis of the picking process of a multi-deep system, when the rack is located at the first position in a lane, it can be directly lifted and then transported to the workstation without rack arrangement. Letting T_{fk} be the expected travel time of the rearrangement for the kth position in a multi-deep lane, the arrangement travel time can be expressed as

$$T_f = \frac{1}{d}\left[0 + T_{f2} + T_{f3} + \cdots T_{fd}\right]. \tag{32}$$

When the target rack is located at the second storage position of a lane, it takes five movements of the rack to turn the target rack to the outside. According to different coordinates and arrangement procedures, the arrangement travel time of the RM machine for the 2-deep lane is presented as

$$T_{f2} = \frac{1}{v}\left[W_l + 2W_l + W_l + (W_l + L_l) + L_l + (W_l + L_l) + W_l + (2W_l + L_l) + (W_l + L_l) + L_l\right]. \quad (33)$$

When the target rack is located at the third storage position in a lane, according to the arrangement procedure, it needs to be moved from the third position to the second position in the lane. Let P_k be the kth position for a multi-deep lane where k is the serial number of the rack position. Therefore, when the target rack is located at the third position of a lane, the rearrangement time of the RM machine, denoted T_{f3}, can be expressed as the sum of two parts, namely the rearrangement time of the RM machine for the second rack position (T_{f2}) and the travel time ($T_f(P_3, P_2)$) from the third to the second rack position:

$$\begin{aligned} T_{f3} &= T_{f2} + T_f(P_3, P_2) \\ &= T_{f2} + \frac{1}{v}\left[\begin{array}{l} W_l + 2W_l + W_l + 2W_l + W_l + (W_l + L_l) + L_l + \\ \quad (W_l + L_l) + W_l + 2W_l + W_l + (2W_l + L_l) + (W_l + L_l) + L_l \end{array}\right] \\ &= T_{f2} + \frac{1}{v}(16W_l + 6L_l) \\ &= \frac{1}{v}(26W_l + 12L_l) \end{aligned} . \quad (34)$$

In the same way, when the target rack is located at the dth position in the d-deep system, the arrangement travel time of the RM machine can be presented as

$$T_{fd} = \frac{1}{v}\left[(3d^2 + d - 4)W_l + (6d - 6)L_l\right]. \quad (35)$$

According to equations (32)–(35), the arrangement travel time of the RM machine can be derived as

$$\begin{aligned} T_f &= \frac{1}{d}\left[T_{f2} + T_{f3} + \cdots T_{fd}\right] \\ &= \frac{1}{d} \cdot \frac{1}{v}\left[(10W_l + 6L_l) + (26W_l + 12L_l) + \cdots + (3d^2 + d - 4)W_l + (6d - 6)L_l\right] \\ &= \frac{1}{d} \cdot \frac{1}{v}\left[\begin{array}{r} \left(3\left(\frac{d(d+1)(2d+1)}{6} - 1\right) + \frac{(2+d)(d-1)}{2} - 4(d-1)\right)W_l + \\ 3(2+d)(d-1) - 6(d-1)L_l \end{array}\right] \\ &= \frac{1}{d} \cdot \frac{1}{v}\left[(d^3 + 2d^2 - 3d)W_l + (3d^2 - 3d)L_l\right] \\ &= \frac{1}{v}\left[(d^2 + 2d - 3)W_l + (3d - 3)L_l\right] \end{aligned} . \quad (36)$$

5.4 Equation of the expected travel time in the multi-deep environment

According to equations (2), (27), (31), and (36), the expected travel time of the RM machine for an operation cycle in the multi-deep system is

$$T_{RMM}^{(d)} = \frac{1}{6vnp}\left[\begin{array}{l} 6nd^2 + \ 16n^2 + 12n - 4 \ \ d - 12n + 3 \ \ pW_l + \\ \qquad 4p^2 + 9pd - 9p - 1 \ \ 2nL_l + \ \ 8n^2 + 18n + 1 \ \ W_a \end{array}\right]. \tag{37}$$

6 Experimental validation and comparison of the analytical model and simulation results

Proposed analytical models are implemented in this section to compare with results of simulation. Six scenarios are calculated and simulated as the background, and relative errors are presented in section 6.2 while the optimal lane depth of the partial experiment scenarios is presented in section 6.3.

6.1 Simulation procedure

The simulation procedure is presented in Fig. 8 according to the assumptions of analytical travel time models.

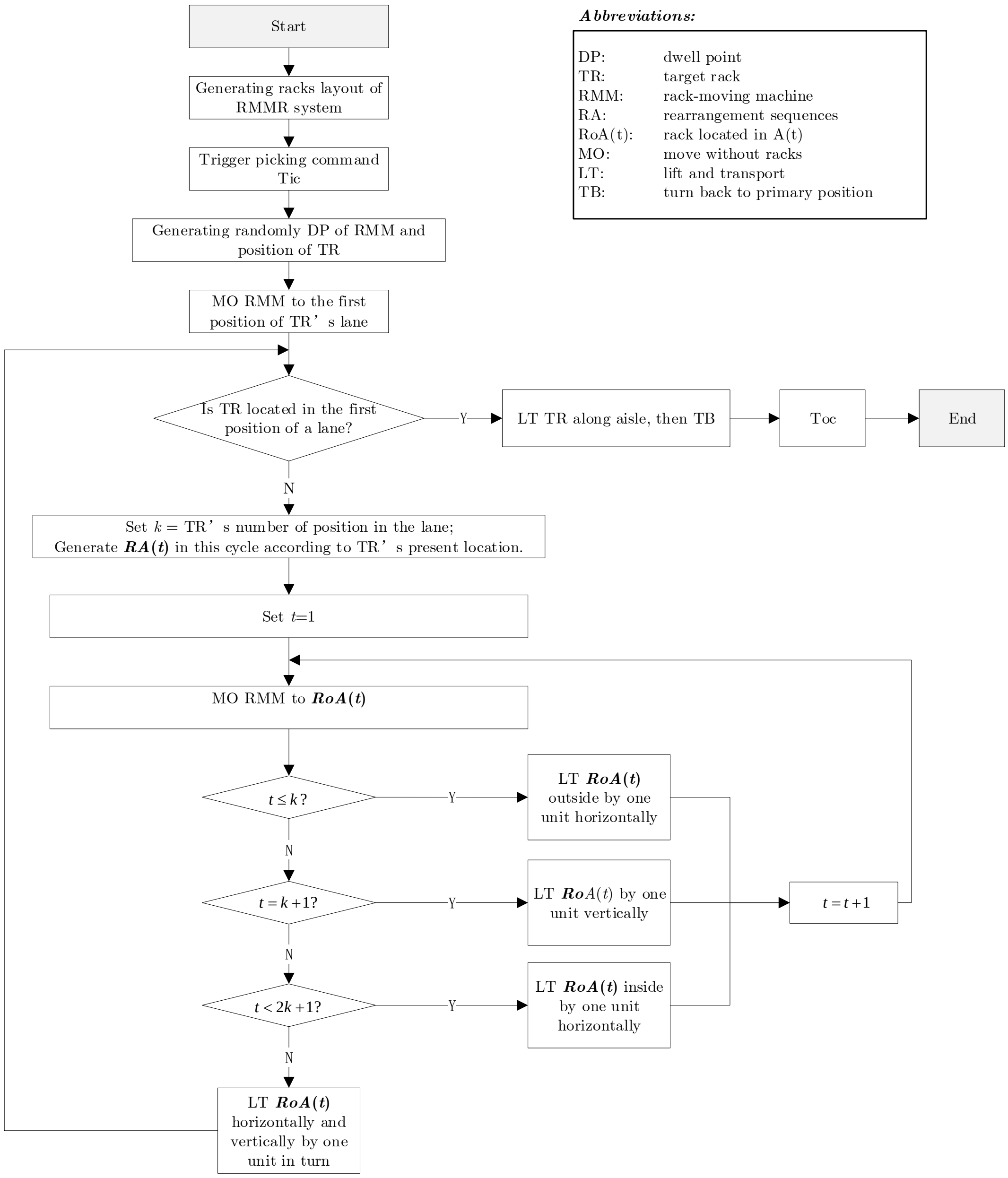


Fig. 8. Simulation flowchart for the command cycle of the RM machine

AnyLogic software is used to simulate a command cycle of the RM machine in various scenarios of different layouts in RMMRs and to validate the accuracy of the analytical model. On the basis of the simulation flowchart of the command cycle of the RM machine, the simulation procedure using AnyLogic involves generation of the initial layout, creation of the picking order, and setting of parameters, variables, and functions.

The simulation parameters are set to the number of vertical aisles (denoted n), the number of rows of racks (denoted p), and the lane depth (denoted d). With different values of simulation variables, six scenarios are constructed for n = 4 or 8 and p = 16, 32 or 64, with each scenario involving four sub-scenarios with different lane depths of 1, 2, 3, and 4, as shown in Table 1.

Table. 1. Various scenarios with different values of n, p and, d

	n	p	d	# of Total Racks
Scenario 1	4	16	1	64
	4	16	2	128
	4	16	3	192
	4	16	4	256
Scenario 2	8	16	1	128
	8	16	2	256
	8	16	3	384
	8	16	4	512
Scenario 3	4	32	1	128
	4	32	2	256
	4	32	3	384
	4	32	4	512
Scenario 4	8	32	1	256
	8	32	2	512
	8	32	3	768
	8	32	4	1024
Scenario 5	4	64	1	256
	4	64	2	512
	4	64	3	768
	4	64	4	1024
Scenario 6	8	64	1	512
	8	64	2	1024
	8	64	3	1536
	8	64	4	2048

6.2 Comparison of the analytical model and simulation results

Suppose that

(1) the travel velocity of the RM machine is a constant $v = 1.5$ m/s;

(2) the width of each aisle is $W_a = 3$ meters; and

(3) for a single rack position, the length L_l is 1.35 meters and width W_l is 1.15 meters.

AnyLogic software is used to simulate 24 classes of scenarios, with 30,000 random experiments for each scenario, which implies the target picking positions are randomly generated 30,000 times. The mean of the simulation results for each sub-scenario is taken as the final results to be compared with the results of the analytical model. Table 2 gives the results of the analytical travel time model and simulations.

Table. 2. Comparison of the results of the travel time model and simulation

		d=1	d=2	d=3	d=4
n=4, p=16	Model Result	33.13	45.46	59.13	74.13
	Simulation Result	33.12	45.52	59.01	74.13
n=8. p=16	Model Result	47.33	66.83	87.67	109.83
	Simulation Result	47.38	66.87	87.41	109.81
n=4, p=32	Model Result	47.35	59.69	73.35	88.35
	Simulation Result	47.30	59.68	73.43	88.19
n=8, p=32	Model Result	61.56	81.06	101.90	124.06
	Simulation Result	61.56	81.02	101.95	124.29

n=4, p=64	Model Result	75.80	88.14	101.80	116.80
	Simulation Result	75.99	88.16	101.94	116.95
n=8, p=64	Model Result	90.01	109.51	130.34	152.51
	Simulation Result	89.98	109.89	130.64	152.45

The relative error (i.e., the difference between results obtained by simulation and those derived using analytical travel time models) can therefore be calculated as

$$\text{relative error}=\frac{\text{simulation value - analytical value}}{\text{analytical value}}. \tag{38}$$

Relative errors of all scenarios are shown in Fig. 9. All relative errors are between -0.35% and 0.3%, which implies that the proposed analytical travel time models are effective.

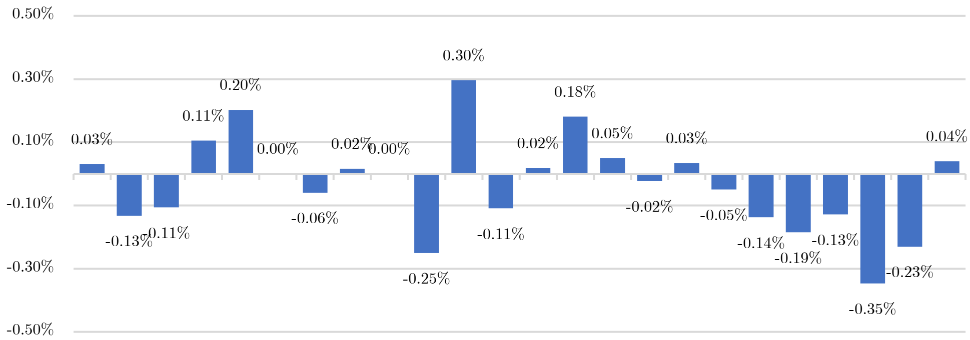


Fig. 9. Relative error (i.e., difference between results obtained by simulation and using the analytical model)

6.3 Optimal lane depth in the partial experiment scenario

Furthermore, five types of layout with 512 racks are constructed according to the simulation scenarios; i.e., two 4-deep layouts of {n = 8, p = 16} and {n = 4, p = 32}, two 2-deep layouts of {n = 8, p = 32} and {n = 4, p = 64}, and one single-deep layout of {n = 6, p = 64}. The results calculated using the analytical multi-deep travel model are shown as Fig. 10. The optimal lane depth selection for the layout of 512 racks is the 2-deep layout with eight vertical aisles and 32 layers, which is presented in Fig. 11(a).

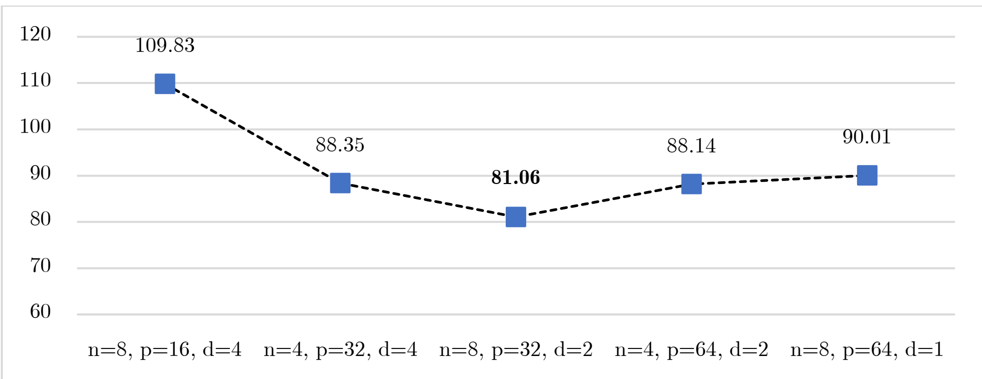
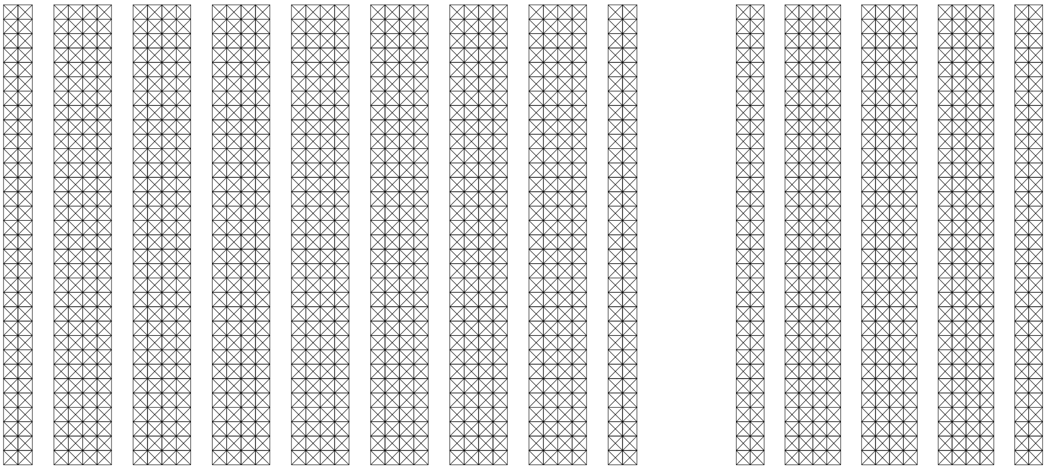


Fig. 10. Comparison of travel times for various depths of 512 racks in the RMMR system



(a) n = 8, p = 32, d = 2, 512 racks (b) n = 4, p = 32, d = 2, 256 racks

Fig. 11. Optimal layouts of 512 racks and 256 racks in the RMMR system

Similarly, there are five types of layout with 256 racks in the proposed simulation scenarios, namely one 4-deep layout of {n = 4, p = 16}, two 2-deep layouts of {n = 8, p = 16} and {n = 4, p = 32}, and two single-deep layouts of {n = 8, p = 32} and {n = 4, p = 64}. The results calculated using the analytical multi-deep travel model are shown as Fig. 12. The optimal lane depth selection for the layout of 256 racks is the 2-deep layout with 4 vertical aisles and 32 layers, which is presented in Fig. 11(b).

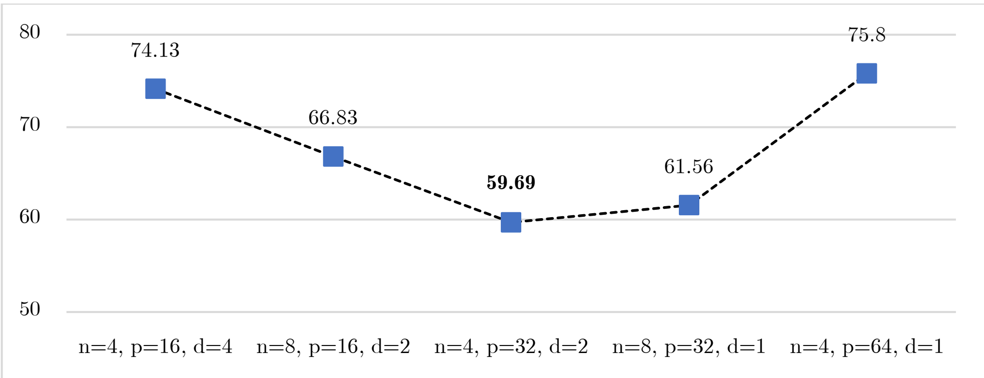


Fig. 12. Comparison of the travel times for various depths of 256 racks in the RMMR system

7 Conclusions

This paper presented original and effective models for the determination of the travel time in single- and multi-deep RMMR systems having a vertical aisle, lane layout, RM machine, and workstation. On the basis of the analysis of the operation command cycle for the RM machine, expected travel time models for single- and multi-deep layouts were proposed and illustrated. Simulations were conducted to validate the proposed models for six scenarios having depth of 1–4 . The simulation results indicate that (1) the absolute difference between the proposed models and AnyLogic simulation (i.e., relative error) is less than 0.35%

and (2) a double-deep lane with a certain number of aisles and layers is optimal. The simulation shows the proposed models are a useful and effective tool with which to estimate the RMMR system performance in terms of the cycle time and provides practical guidelines for the design of a storage system in terms of the numbers of aisles and layers and lane depth.

The present study has limitations despite the breakthrough it makes. Future work will focus on the following. (1) The optimal 2-deep lane was found using the proposed rearrangement strategy; we can therefore obtain a new optimal lane depth and layout if the rearrangement strategy is improved. (2) Space utilization is another factor used to analyze the performance of the warehousing system. It can thus be considered simultaneously, and a multi-objective optimization model can be developed to analyze the tradeoff between the space utilization and travel time. (3) If the horizontal aisle is considered, the layout is modular and compact, which implies a new dual-dimension lane depth (with horizontal and vertical directions). The optimal layout in various conditions and from various perspectives will therefore be central to further analysis and research.

Acknowledgments

This work was supported by the National Social Science Foundation of China (Grant No. 16CGL018), JSPS KAKENHI (Grant No. 15K03739/18H00883), and the Research Institute for Innovation Management at Hosei University. We thank Glenn Pennycook, MSc, from Edanz Group (www.edanzediting.com/ac) for editing a draft of this manuscript.

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