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Theoretical Relation Between Expected Option Returns and Pricing Kernel

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Theoretical Relation Between Expected Option Returns and Pricing Kernel*

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Abstract

This research is aimed at examining the theoretical relations between expected option returns and a pricing kernel. Under mild assumptions, it is demonstrated that the condition of the tail of the pricing kernel slope characterizes the slope and curvature of the expected option returns. This study also determines the threshold levels of the pricing kernel for each case in which the expected call returns are negative or the expected put returns exceed the risk-free rate, thereby violating the classical asset pricing theory. The results of this study are more comprehensive and informative than those of previous works.

Keywords: expected option return, pricing kernel, static replication method
Classification codes: G12, G13

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1 Introduction

Understanding the relation between expected option returns and a pricing kernel is essential to characterizing the option returns and pricing kernel. The classical asset pricing theory states that the pricing kernel reduces monotonically in aggregate consumption. Based on this, Coval and Shumway (2001) demonstrated that the expected option returns increase with the strike price, the expected call returns exceed the expected underlying asset return, and the expected put returns remain below a risk-free rate.

However, the extant empirical studies contradict this with puzzling evidence. For example, Jackwerth (2000, 2004) and Yamazaki (2022) estimated that pricing kernels were tilde-shaped employing the S&P 500 index data, while Bakshi et al. (2010) and Christoffersen et al. (2013) argued that empirical pricing kernels were U-shaped (see Cuesdeanu and Jackwerth (2018) for existing studies on pricing kernels). Concurrently, Bakshi et al. (2010) illustrated how the average returns of deep out-of-the-money (OTM) index call options in major international stock markets decrease with the strike price and are negative. Coval and Shumway (2001), Bondarenko (2014), and others reported that the average put returns are too low to be consistent with standard models.

This research examines the theoretical relations between expected option returns and the shape of a pricing kernel in a model-free setup under several mild assumptions, and the results reveal the implications of the expected option returns from the slope condition for the pricing kernel. These implications are more informative than those of past studies.

This study demonstrated that the downward slope of the right tail of the pricing kernel indicated that the expected call returns were convex and increased with the strike price, while the upward slope indicated that they were concave and decreased with the strike price. Meanwhile, the downward slope of the left tail of the pricing kernel indicated that the expected put returns were concave and increasing, while its upward slope indicated that it was convex and decreasing. Further, we demonstrated that the expected call returns were negative when the pricing kernel increased and was greater than 1. Furthermore, the expected put returns exceeded the risk-free rate when the pricing kernel increased and was less than the discount bond price.

Our model-free argument employed a static replication method that differs from those of previous studies. Static replication methods have been applied to hedging for derivative products (see Carr et al. (1998), Fink (2003), Nalholm and Poulsen (2006), Chung and Shih (2009), and Takahashi and Yamazaki (2009a, 2009b). Dissimilar to such hedging studies, the argument here applied the static replication method for obtaining the representation of an expected option payoff as the price of a static option portfolio. Thus, concrete expressions of the expected option returns were obtained. The expressions favored the analysis because it could argue the theoretical relations from the perspective of the geometry of option price functions. Contrarily, the previous studies on option returns, such as the studies of Coval and Shumway (2001), Bakshi et al. (2010), and Chaudhuri and Schroder (2015), developed more abstract arguments.

The aforementioned related studies all adopted model-free approaches, and their results were compared with those reported here in Section 3. Model-specific approaches for analyzing option returns have been studied by Broadie et al. (2009), Christoffersen et al. (2013), and Yamazaki (2018, 2020) (See Appendix A of Broadie et al. (2009) for the previous research on option returns before 2009).

The remainder of this paper proceeds, as follows: Section 2 presents the model-free setup under several mild assumptions. Section 3 presents the results and compares them with those of past theoretical analyses. Section 4 concludes the study, and the appendix states all the proofs and some lemmas employed.

2 Setup

This section presents the model-free setup for describing the expected option returns and pricing kernel. Assume that there are no arbitrage opportunities and that all the plain vanilla options on an asset are tradable without transaction costs. Consider a fixed period, $[0, T]$, and let S_t be the asset price at time $t \in [0, T]$.

2.1 Expected payoffs and option price functions

The expected payoffs of the call and put options with the strike price K, and maturity T, are given, as follows:

$$\mathbb{E}\left[(S_T - K)_+\right] \qquad \text{and} \qquad \mathbb{E}\left[(K - S_T)_+\right], \tag{2.1}$$

respectively. Notably, $\mathbb{E}[\,\cdot\,]$ is assumed as the expectation operator under the subjective probability measure, $\mathbb{P}$, of the representative investor. The prices of the call and put options will be expressed, as follows:

$$C(K) := \frac{1}{R_f}\mathbb{E}_*\left[(S_T - K)_+\right] \qquad \text{and} \qquad P(K) := \frac{1}{R_f}\mathbb{E}_*\left[(K - S_T)_+\right], \tag{2.2}$$

respectively, where $\mathbb{E}_*[\,\cdot\,]$ denotes the expectation operator under the risk-neutral probability measure, $\mathbb{Q}$, and R_f is the gross return on a risk-free asset during the period.

Let F be the forward price of the asset expiring at time T. Mild assumptions were imposed on the option price functions, as follows:

Assumption 1 *The call price function, $C(x)$, exhibits the following properties:*

1. *$C(x)$ is a strictly convex and decreasing function on $[F, \infty)$.*
2. *$C(x)$ is differentiable on $[F, \infty)$.*
3. *$C(x)$ converges rapidly to zero as x approaches infinity.*

Assumption 2 *The put price function, $P(x)$, exhibits the following properties:*

1. *$P(x)$ is a strictly convex and increasing function on $(0, F]$.*
2. *$P(x)$ is differentiable on $(0, F]$.*
3. *$P(x)$ converges rapidly to zero as x approaches zero.*

This research addresses the expected rate of the returns on the OTM options, defined, as follows:

$$ECR(K) := \frac{\mathbb{E}\left[(S_T - K)_+\right]}{C(K)} - 1 \qquad \text{and} \qquad EPR(K) := \frac{\mathbb{E}\left[(K - S_T)_+\right]}{P(K)} - 1. \tag{2.3}$$

According to Coval and Shumway (2001), employing the expected log-returns of a plain vanilla option is quite challenging because the log returns diverge and become worthless when the option expires. Thus, the expected rate of returns rather than expected log-returns is employed.

2.2 Projected pricing kernel and reciprocal kernel

Let $\mathcal{M}$ be a strictly positive random variable, such that the price of an arbitrary payoff X paid at time T is given by the following:

$$\mathbb{E}\left[\mathcal{M}X\right]. \tag{2.4}$$

Namely, $\mathcal{M}$ is a pricing kernel[1]. The arbitrage-free condition ensures the existence of a strictly positive pricing kernel. The projection of the pricing kernel on S_T is defined as

$$\mathcal{M}(S_T) := \mathbb{E}\left[\mathcal{M} \mid S_T\right]. \tag{2.5}$$

Assume that the projected pricing kernel, $\mathcal{M}(x)$, was twice differentiable for any $x > 0$.

The shape of $\mathcal{M}(x)$ was largely controversial. The classical asset pricing theory states that the pricing kernel projected onto the market portfolio decreases monotonically. However, several researchers have reported that such a theoretical result is inconsistent with empirical analyses. For example, via an empirical estimation that was based on the S&P 500 index, Jackwerth (2000, 2004) and Yamazaki (2022) obtained tilde-shaped pricing kernels. However, Bakshi et al. (2010) and Christoffersen et al. (2013) proved that empirical pricing kernels are U-shaped. Concurrently, Chaudhuri and Schroder (2015) reported that empirical pricing kernels that were projected onto individual stocks exhibited downward slopes in the whole range. This scope of this research does not cover the appropriate shape of a pricing kernel because it may depend on the choice of an asset onto which the pricing kernel is projected.

Next, the reciprocal of $\mathcal{M}(x)$, which Yamazaki (2022) calls the *reciprocal kernel*, was defined, as follows:

$$g(x) := 1/\mathcal{M}(x). \tag{2.6}$$

The reciprocal kernel is equivalent to $\mathcal{M}(x)$ in effect. However, it is convenient to apply the reciprocal kernel, $g(x)$, rather $\mathcal{M}(x)$. Notably, the reciprocal kernel was strictly positive and two times differentiable under our assumptions.

3 Results

3.1 Expected call returns

This subsection presents the theoretical results of the relation between the expected call returns and pricing kernel, and the proofs are presented in the Appendix.

Proposition 1 *For any $K > F$, the expected call return is represented, as follows:*

$$ECR(K) = g(K) + \frac{1}{C(K)} \int_K^\infty \left[(x-K)g''(x) + 2g'(x)\right] C(x)dx - 1. \tag{3.1}$$

Another representation is given by the following:

$$ECR(K) = -\frac{1}{C(K)} \int_K^\infty \left[(x-K)g'(x) + g(x)\right] C'(x)dx - 1. \tag{3.2}$$

[1]Employing the pricing kernel, the option prices can also be written, as follows:

$$C(K) = \mathbb{E}\left[\mathcal{M}(S_T - K)_+\right] \qquad \text{and} \qquad P(K) = \mathbb{E}\left[\mathcal{M}(K - S_T)_+\right].$$

Additionally, it is critical to emphasize that the expectation operators in the above option pricing equations are under the representative investor's subjective probability measure, $\mathbb{P}$.

Proposition 1 avails more explicit representations of the expected call returns than past studies. For example, Representation (3.1) indicates that the expected payoff of a call option was equal to the price of a static option portfolio. This portfolio comprises a long position in $g(K)$ units of the call option itself and a long position in $[(x-K)g''(x)+2g'(x)]\,dx$ units of the call options, with all strikes, x, larger than K. Representation (3.2) states that the expected call returns can be expressed without the curvature of the reciprocal kernel, indicating that any information about the curvature of the pricing kernel is unnecessary for determining the expected call returns. These representations produce the following theorems:

Theorem 1 *(a) If $L > F$ exists such that $\mathcal{M}'(x) < 0$ for all $x \geq L$, the expected call return, $ECR(K)$, will be convex and increasing in $K \in [L, \infty)$.*
(b) If $L > F$ exists such that $\mathcal{M}'(x) > 0$ for all $x \geq L$, $ECR(K)$ will be concave and decreasing in $K \in [L, \infty)$.

Theorem 2 *If $L > F$ exists such that $\mathcal{M}(L) > 1$ and $\mathcal{M}'(x) > 0$ for all $x \geq L$, $ECR(K)$ for $K \geq L$ will be negative.*

Table 1 summarizes the theoretical implications of the shape of projected price kernels for the expected call returns. The terminology, "locally," in the table denotes the right tail of a projected pricing kernel. Notably, all the implications were derived from model-free setups.

Coval and Shumway (2001) considered a monotonically decreasing pricing kernel that is consistent with the classical asset pricing theory. Bakshi et al. (2010) assumed a U-shaped pricing kernel, which was projected onto the log-return of an asset price rather than onto the asset price. This pricing kernel exhibited an upward slope and positive convexity in the right region. Moreover, Theorem 1, as well as Chaudhuri and Schroder (2015) only required the local slope conditions on projected pricing kernels. Theorem 1 deals with the downward and upward cases. Notably, Theorem 1 offers information on the slope of the expected call return and its curvature.

Coval and Shumway (2001) proved that expected call returns are always positive when the pricing kernel decreases monotonically. Conversely, Bakshi et al. (2010) demonstrated that the expected returns on deep OTM call options might be negative if the pricing kernel was U-shaped. Similarly, Theorem 2 indicates the possibility of obtaining negative expected call returns. Furthermore, Theorem 2 states that the threshold level of the projected pricing kernel to produce negative expected call returns was 1.

3.2 Expected put returns

In this subsection, the corresponding results of the expected put returns are presented. The proofs are similar to the case of the expected call returns (See Appendix for details).

Proposition 2 *For any $K < F$, the expected put return is represented, as follows:*

$$EPR(K) = g(K) + \frac{1}{P(K)} \int_0^K \left[(K-x)g''(x) - 2g'(x)\right] P(x)dx - 1. \tag{3.3}$$

Another representation is given, as follows:

$$EPR(K) = -\frac{1}{P(K)} \int_0^K \left[(K-x)g'(x) - g(x)\right] P'(x)dx - 1. \tag{3.4}$$

Table 1. Implications of the pricing kernel for the expected call returns

	Projected pricing kernel	Expected call return
Coval and Shumway (2001)	decreasing monotonically	increasing positive
Bakshi et al. (2010)	U-shaped	decreasing negative in deep OTM
Chaudhuri and Schroder (2015)	locally decreasing	increasing
Theorem 1	decreasing locally locally increasing	convex and increasing concave and decreasing
Theorem 2	increasing locally and greater than 1	negative

Theorem 3 *(a) If $L < F$ exists such that $\mathcal{M}'(x) < 0$ for all $0 < x \leq L$, $EPR(K)$ will be concave and increasing in $K \in (0, L]$.*
(b) If $L < F$ exists such that $\mathcal{M}'(x) > 0$ for all $0 < x \leq L$, $EPR(K)$ will be convex and decreasing in $K \in (0, L]$.

Theorem 4 *If $L < F$ exists, such that $\mathcal{M}(L) < 1/R_f$ and $\mathcal{M}'(x) > 0$ for all $0 < x \leq L$, $EPR(K)$ for $K \leq L$ will exceed the risk-free rate.*

Table 2 summarizes the theoretical implications of the shape of projected price kernels for the expected put returns. The terminology, "locally," in the table denotes the left tail of a projected pricing kernel. However, Bakshi et al. (2010) did not consider the put case.

Theorem 3 also offered information about the slope and curvature of the expected put returns from the slope of a projected pricing kernel in the left region. Coval and Shumway (2001) showed that the risk-free rate is the upper bound of expected put returns when the pricing kernel monotonically decreases. Meanwhile, Theorem 4 proves that the expected put returns exceeded the risk-free rate if the projected pricing kernel was less than the discount bond price and increased in the left region.

Table 2. Implication of the pricing kernel for the expected put return

	Projected pricing kernel	Expected put return
Coval and Shumway (2001)	decreasing monotonically	increasing less than risk-free rate
Chaudhuri and Schroder (2015)	decreasing locally	increasing
Theorem 3	locally decreasing locally increasing	concave and increasing convex and decreasing
Theorem 4	locally increasing and less than discount bond price	larger than risk-free rate

4 Conclusion

This research presents the theoretical implications of the shape of a pricing kernel for the expected option returns under a model-free setup. The slope condition of the tails of the projected pricing kernel determines the sign of the slope and convexity of the expected option returns. Furthermore, we specify the threshold levels of the pricing kernel for each case where the expected call returns are negative or where the expected put returns exceed the risk-free rate, in which the classical asset pricing theory is violated. The implications of this research are more comprehensive and informative than those of the existing studies.

This research mainly focuses on only the theoretical aspects. Thus, a conceivable future research topic will focus on an empirical investigation. For example, the static replication method for obtaining the representations of the expected option payoffs might offer a new model-free approach for testing the option returns and shape of the pricing kernels.

A Proofs

The following lemma is simple but crucial to the derivation of the results. See Appendix A.1 of Yamazaki (2022) for the proof.

Lemma 1 (Yamazaki, 2022) *Let $H(S_T)$ be an arbitrary payoff depending on the time-T price of the underlying asset. Thus, the expected payoff of $H(S_T)$ will be equal to the price of the product of the reciprocal kernel $g(S_T)$ and $H(S_T)$. Namely,*

$$\mathbb{E}\left[H(S_T)\right] = \frac{1}{R_f}\mathbb{E}_*\left[g(S_T)H(S_T)\right].$$

The convexity of the option pricing functions yields the two following lemmas: They are applied in the proofs of Theorems 1 and 3.

Lemma 2 *Let $K > F$. Define*

$$\xi(x) := C(K)C'(x) - C'(K)C(x) + (x-K)C'(K)C'(x). \tag{A.1}$$

Then $\xi(x) > 0$ for any $x > K$.

Proof of Lemma 2: (A.1) can be rewritten as

$$\xi(x) = \left|C'(K)\right| C(x) - \left|C'(x)\right| l(x),$$

where $l(x) := C(K) + (x-K)C'(K)$. Notably, $l(x)$ is the tangent line of curve $y = C(x)$ at a point, $x = K$. Since $C(x)$ is a strictly convex and decreasing function, $C(x) > l(x)$ and $|C'(K)| > |C'(x)|$ for any $x > K$. □

Lemma 3 *Let $0 < K < F$. Define*

$$\zeta(x) := P(K)P'(x) - P'(K)P(x) - (K-x)P'(K)P'(x). \tag{A.2}$$

Then $\zeta(x) < 0$ for any $0 < x < K$.

Proof of Lemma 3: (A.2) can be rewritten as

$$\zeta(x) = P'(x)m(x) - P'(K)P(x),$$

where $m(x) := P(K) + (x-K)P'(K)$. Notably, $m(x)$ is the tangent line of curve $y = P(x)$ at a point, $x = K$. Since $P(x)$ is a strictly convex and increasing function, $P(x) > m(x)$ and $P'(K) > P'(x)$ for any $0 < x < K$. □

A.1 Proof of the Proposition 1

Set $H(S_T) = (S_T - K)_+$ and $f(S_T) := g(S_T)H(S_T)$. According to Carr and Madan (1998), the following will be obtained:

$$f(S_T) = f(F) + f'(F)(S_T - F) + \int_0^F f''(x)(x - S_T)_+ dx + \int_F^\infty f''(x)(S_T - x)_+ dx.$$

Employing Lemma 1 and $K > F$, the expected payoff of $H(S_T)$ can be expressed, as follows:

$$\mathbb{E}\left[H(S_T)\right] = \frac{1}{R_f}\mathbb{E}_*\left[f(S_T)\right] = \int_0^F f''(x)P(x)dx + \int_F^\infty f''(x)C(x)dx. \quad \text{(A.3)}$$

Note that,

$$f''(x) = g''(x)(x-K)_+ + 2g'(x)\mathbf{1}_{\{x>K\}} + g(x)\delta(x-K),$$

where $\delta(x)$ is the Dirac delta function. Therefore, the following is obtained:

$$\begin{aligned}\int_0^F f''(x)P(x)dx &= \int_0^F g''(x)(x-K)_+P(x)dx + 2\int_0^F g'(x)\mathbf{1}_{\{x>K\}}P(x)dx \\ &\quad + \int_0^F g(x)\delta(x-K)P(x)dx = 0, \quad \text{(A.4)}\end{aligned}$$

and

$$\begin{aligned}\int_F^\infty f''(x)C(x)dx &= \int_K^\infty g''(x)(x-K)C(x)dx + 2\int_K^\infty g'(x)C(x)dx \\ &\quad + \int_F^\infty g(x)\delta(x-K)C(x)dx \\ &= \int_K^\infty \left[g''(x)(x-K) + 2g'(x)\right]C(x)dx + g(K)C(K). \quad \text{(A.5)}\end{aligned}$$

Substituting (A.4) and (A.5) into (A.3) yields (3.1). Meanwhile, using the integration by parts formula, we have

$$\int_F^\infty f''(x)C(x)dx = -\int_K^\infty \left[g'(x)(x-K) + g(x)\right]C'(x)dx. \quad \text{(A.6)}$$

Substituting (A.4) and (A.6) into (A.3) yields (3.2). □

A.2 Proof of Theorem 1

Proposition 1 yields

$$ECR'(K) = \frac{1}{C(K)^2}\int_K^\infty \xi(x)g'(x)dx,$$

and

$$ECR''(K) = -2\frac{C'(K)}{C(K)^3}\int_K^\infty \xi(x)g'(x)dx,$$

where $\xi(x)$ is defined by (A.1). Recall that $C(K) > 0$ and $C'(K) < 0$. Lemma 2 reveals that $\xi(x) > 0$ for any $x > K > F$. Notably, $g'(x) < 0$ if and only if $\mathcal{M}'(x) > 0$. □

A.3 Proof of Theorem 2

Fix $K \geq L$. Suppose that $\mathcal{M}(L) > 1$ and $\mathcal{M}'(x) > 0$ for all $x \geq L$. This condition indicates that $g(K) < 1$ and $g'(x) < 0$ for all $x \geq K$. Define

$$v(x) := (x - K)g'(x) + g(x). \tag{A.7}$$

Note that $v(K) = g(K) < 1$. Since $(x - K)g'(x) \leq 0$ and $g(x)$ is a decreasing function on $[K, \infty)$, we have $v(x) \leq v(K) < 1$. Therefore, Proposition 1 yields the following inequality:

$$ECR(K) = -\frac{1}{C(K)} \int_K^\infty v(x)C'(x)dx - 1 < -\frac{1}{C(K)} \int_K^\infty C'(x)dx - 1 = 0.$$

□

A.4 Proof of Proposition 2

Set $H(S_T) = (K - K)_+$ and $h(S_T) := g(S_T)H(S_T)$. Similar to the proof of Proposition 1, the expected payoff of $H(S_T)$ for $K < F$ can be expressed, as follows:

$$\mathbb{E}\left[H(S_T)\right] = \frac{1}{R_f}\mathbb{E}_*\left[h(S_T)\right] = \int_0^F h''(x)P(x)dx + \int_F^\infty h''(x)C(x)dx. \tag{A.8}$$

Notably,

$$h''(x) = g''(x)(K - x)_+ - 2g'(x)\mathbf{1}_{\{x<K\}} + g(x)\delta(x - K).$$

Therefore, the following is obtained:

$$\begin{aligned}
\int_0^F h''(x)P(x)dx &= \int_0^K g''(x)(K - x)P(x)dx - 2\int_0^K g'(x)P(x)dx \\
&\quad + \int_0^F g(x)\delta(x - K)P(x)dx \\
&= \int_0^K \left[g''(x)(K - x) - 2g'(x)\right] P(x)dx + g(K)P(K).
\end{aligned} \tag{A.9}$$

and

$$\begin{aligned}
\int_F^\infty h''(x)C(x)dx &= \int_F^\infty g''(x)(K - x)_+C(x)dx - 2\int_F^\infty g'(x)\mathbf{1}_{\{x<K\}}C(x)dx \\
&\quad + \int_F^\infty g(x)\delta(x - K)C(x)dx = 0.
\end{aligned} \tag{A.10}$$

Substituting (A.9) and (A.10) into (A.8) yields (3.3). Meanwhile, using the integration by parts formula, we have

$$\int_0^F h''(x)P(x)dx = -\int_0^K \left[g'(x)(K - x) - g(x)\right] P'(x)dx. \tag{A.11}$$

Substituting (A.10) and (A.11) into (A.8) yields (3.4). □

A.5 Proof of Theorem 3

Proposition 2 yields

$$EPR'(K) = -\frac{1}{P(K)^2}\int_0^K \zeta(x)g'(x)dx,$$

and

$$EPR''(K) = 2\frac{P'(K)}{P(K)^3}\int_0^K \zeta(x)g'(x)dx,$$

where $\zeta(x)$ is defined by (A.2). Recall that $P(K) > 0$ and $P'(K) > 0$. Lemma 3 reveals that $\zeta(x) < 0$ for any $x < K < F$. Notably, $g'(x) < 0$ if and only if $\mathcal{M}'(x) > 0$. □

A.6 Proof of Theorem 4

Fix $K \leq L$. Suppose that $\mathcal{M}(L) < 1/R_f$ and $\mathcal{M}'(x) > 0$ for all $x \geq L$. This condition indicates that $g(K) > R_f$ and $g'(x) < 0$ for all $x \leq K$. In this case, $v(K) = g(K) > R_f$, where $v(x)$ is defined by (A.7). Since $(x - K)g'(x) \geq 0$ and $g(x)$ is a decreasing function on $(0, K]$, we have $v(x) \geq v(K) > R_f$. Therefore, Proposition 2 yields the following inequality:

$$EPR(K) = \frac{1}{P(K)}\int_0^K v(x)P'(x)dx - 1 > \frac{1}{P(K)}\int_0^K R_f P'(x)dx - 1 = R_f - 1.$$

□

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